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by A. Baz' and Ya. Smorodinskiy

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PART I. THEORY

Chapter I. THE ISOTOPIC SPIN OF NUCLEONS

1. Introduction

There are many grounds for the belief that the nuclear forces operating among the nucleons possess a charge independence. That means that the interaction of three possible pairs of nucleons -- proton-proton, neutron-neutron and proton-neutron -- found in the same states (in point of the relationship between the wave functions and coordinates and spins) are identical with one another.

However, despite the wide acceptance of the hypothesis of charge independence in nuclear physics, there is at present no direct proof of its validity.

Actually, the interaction of two nucleons was studied only in the scattering of neutrons by protons and protons by protons. It is well known, however, that in the investigated energy region (approximately up to 10 Mev) a scattering connected with nuclear forces occurs only in $l = 0$ states (S-state). An analysis of these experiments merely justified the conclusion that neutron-proton and proton-proton interactions are similar only in ^1S state (it may be recalled that, in view of the Pauli principle, two protons cannot be found in a ^3S-state). The part played by $l > 0$ states in scattering is so insignificant that it is impossible to reach any conclusion on interaction in such states in the low-energy region (up to $\sim$ 10 Mev). The high-energy region, which is dealt with in numerous publications, is a separate problem connected in particular with the charge independence of nucleon interaction with π-mesons (as well as other mesons which may possibly play an important part in the interaction between nucleons). Incidentally, there is no definite proof of charge independence even in the high-energy region, even though all the available experiments do not contradict that hypothesis.

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It is very important therefore to examine the spectral structure of the light nuclei from the point of view of their inherent tendencies resulting from the acceptance of the hypothesis of charge independence. As it turns out, it is the structure of the nuclear spectra itself that provides the most convincing proof of the validity of the hypothesis under consideration, at least in the region of not very high energy.

Our task in this review is to show which general properties of the spectra are connected with the hypothesis of charge independence, and what conclusions may be made about the accuracy of that hypothesis.

The review consists of two parts. In the first part we shall outline the basic physical ideas used as a basis for the development of the isotopic spin theory. Here we shall avoid complex mathematical problems and confine ourselves to less rigid but clear considerations. The theoretical problems are dealt with in the recent publications by Shapiro[1] and Zel'tser[2], and we refer the reader to them.

The second part contains a description of the levels of light nuclei and an analysis of their isotopic spin. Many data on the levels were taken from the two publications on light nuclei by Ajzenberg and Lauritsen[3] and Endta and Kluyver[4].

2. The Quantum Characteristics of Nuclear Levels.

As is known, each level of the quantum system (particularly of the nucleus) is characterized by a set of quantum numbers. These numbers are connected with the various properties of the system's symmetry; there are two types of quantum numbers, exact and approximate, depending on whether the symmetry under consideration is exact or not. The exact numbers, such as the energy of the system and the angular momentum, are fully preserved in any processes occurring in the system; the approximate numbers, generally speaking, may not be preserved but the processes connected with their non-conservation have a considerably lesser probability. The latter may consist of orbital and spin quantum numbers in atoms whose conservation is valid only within the framework of the L-S coupling scheme. In addition to the abovementioned exact quantum numbers, energy E and spin I, the system levels are characterized also by parity P arising from the invariant properties of the system during the reflection of all coordinates in the origin of the coordinates.

It should be recalled that since a twice-repeated reflection is an identical operation, the wave function of the system may either change its sign or remain invariable after a single reflection. In the first case the system is referred to as "odd", and in the second as "even".

These three numbers, E, I and P, exhaust the exact quantum numbers.

We shall not proceed to the "inexact" quantum numbers. The introduction of such numbers is connected with concrete assumptions regarding the properties of the nucleon system. The source of inaccuracy of quantum numbers lies in the approximate nature of these assumptions.

Thus a nuclear shell model with a j-j coupling is conducive to the emergence of an entire series of inaccurate quantum numbers, moments and parities (or complete and orbital moments, which is the same) of separate nucleons in a nucleus. The inaccuracy of such quantum numbers is obvious: it is determined by the disregard of the interaction between nucleons.

Similarly inaccurate is the concept of the isotopic spin.

This concept originated in connection with the hypothesis of the charge independence of nuclear forces under consideration. Such a hypothesis amounts to the requirement that the Hamiltonian (or Lagrange) function of the system remain constant when any proton is replaced by a neutron or, conversely, when any neutron is replaced by a proton. This means, in particular, that the Hamiltonian function of the system must be symmetrical in relation to the simultaneous permutation of the coordinates and spins of any particles.

The result of the invariance of the Hamiltonian system in relation to any permutation is that certain conditions are imposed on the wave function of the nucleus. Only in the simplest two-particle system are these conditions conducive to the symmetry or antisymmetry of the wave function. Generally, their formulation is more complicated. In terse and clear language these conditions may be described as the theory of the isotopic spin, and the charge independence as the law of the isotopic spin conservation.

3. The Isotopic Spin of Nucleons and the Nucleon System

Within the framework of the hypothesis of charge independence, the neutron and proton are considered as two charge states of the same particle, the nucleon. And such a neutron-proton unification into a single particle facilitates a concise formulation of all the consequences flowing from the hypothesis of charge independence; moreover, this method is very convenient for the classification of the states of a system consisting of neutrons and protons.

An appropriate apparatus should be constructed as follows. According to the above, the nucleon should be described by a two-component wave function which may be recorded in the form of a column. In this recording, the proton and neutron states of the nucleon are represented, respectively, as

$$\psi_p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi_n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

We shall introduce operator τ_1 which converts the neutron to a proton. According to the definition, τ_1 should possess the property that

$$\tau_1\psi_n = \psi_p; \quad \tau_1\psi_p = 0.$$

It is easy to see that this operator can be represented as follows:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

Similarly, the operator τ_2 possessing the following property

$$\tau_2\psi_p = \psi_n; \quad \tau_2\psi_n = 0,$$

may be recorded as

$$\tau_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

We shall further introduce operators $\tau_\xi, \tau_\eta, \tau_\zeta$ according to the following equalities:

$$\tau_\xi = (\tau_1 + \tau_2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \tau_\eta = i(\tau_2 - \tau_1) = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \ \tau_\zeta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Thus defined, the operators $\tau_\xi, \tau_\eta, \tau_\zeta$ coincide with the Pauli matrices of the spin theory and, consequently, possess the same formal properties as the latter. In particular, it is easy to verify that operator $^1/_2\tau_\zeta$ produces the following effect on the wave functions of the neutron and proton:

$$^1/_2\tau_\zeta\psi_p = {}^1/_2\psi_p; \ ^1/_2\tau_\zeta\psi_n = -{}^1/_2\psi_n.$$

All the correlations in the theory of the isotopic spin are identical with the analogical correlations of the nonrelativistic theory of the spin. The proton state of a nucleon plays the role of a state with a spin projection ½, a neutron state corresponds to a state with a spin projection ½, while operators $^1/_2\tau_\xi$, $^1/_2\tau_\eta$ и $^1/_2\tau_\zeta$. play the part of projection spin operators on the Cartesian coordinate axes. Hence the reference to the operators $^1/_2\tau_\xi$, $^1/_2\tau_\eta$ & $^1/_2\tau_\zeta$ as operator projections of the isotopic nucleon spin, and the two nucleon charge states are considered as states with different projections of the isotopic nucleon spin $½\tau$ on the ζ-axis.

To pursue the analogy between the ordinary and isotopic spin still further, we shall formally introduce a three-dimensional isotopic space in which τ_ξ, τ_η and τ_ζ can be considered as components of vector operator τ. The reason for introducing an isotopic space is that operator $½\tau$ thereby acquires the obvious significance of an angular momentum operator in this space, the nucleon should be considered as a particle with an isotopic spin ½, and the classification of the states of the system consisting of neutrons and protons is reduced to the well known method of classifying the states of identical particles with spin ½ which occurs in the theory of the electron atom shell.

Inasmuch as the interactions (pp), (pn) and (nn) are equal, according to the hypothesis of the charge independence, the neutrons and protons within a nucleus differ from one another only by the Pauli principle which forbids such states of a system whereby two neutrons or two protons are in the same quantum state. On the other hand, the Hamiltonian of the system is symmetrical in relation to the permutation of the spins and space coordinates of any two particles. If the neutrons and protons are therefore considered as different states of the same particle, the nucleon, it is obvious that the charge independence requires the Hamiltonian of the nucleon system to be invariant in relation to the permutation of all the five coordinates ($xyzs_z\tau_\zeta$) of any two nucleons. Hence the direct corollary that the wave function of any such system must either remain unchanged by the permutation of the coordinates of any two nucleons (symmetrical wave function) or change its sign (antisymmetrical wave function). It is known, however, that neutrons and protons, taken separately, conform to the Pauli principle, that is the wave function of the system changes its sign during the permutation of the space coordinates and spins of two neutrons or two protons. As τ_ζ remains unchanged by such a permutation ($\frac{1}{2}\tau_\zeta = -\frac{1}{2}$ for all neutrons and $\frac{1}{2}\tau_\zeta = \frac{1}{2}$ for all protons), this permutation can be considered as a permutation of all the five coordinates of two nucleons. But if the wave function changes its sign in the permutation of all the five coordinates of even a single pair of nucleons, it can easily be shown that it must be antisymmetrical in relation to the permutation of all the five coordinates of any pair of nucleons. This assertion is usually called the generalized Pauli principle.

Thus on the assumption of the charge independence of nuclear forces, a system consisting of neutrons and protons may be considered as a system of identical particles, nucleons (with a charge degree of freedom), governed by the Fermi statistics.

The question further arises as to the classification of the energy levels of such a system. In this connection, we shall introduce the concept of the isotopic spin of the nucleon system. The isotopic spin of the nucleon system T is defined as the sum of isotopic spins

$$\mathbf{T} = \sum_{i=1}^{N} {}^1/_2\tau^{(i)},$$

where all the spins are summed up. It is clear that operator T, thus defined, is a vector in the isotopic space and, just like $\frac{1}{2}\tau$, possesses all the angular momentum properties. In particular, the usual rules governing the addition of moments* are applicable to the addition of isotopic nucleon spins; for example, in the case of a system of two nucleons, T may assume the values 0 and 1, in a three-nucleon system ½ and 3/2, in a four-nucleon system 0.1 and 2, etc. The physical meaning of states with different isotopic spins can readily be understood. We should note in this connection that, according to the definition of T as a vector, T is at any rate not smaller than its projection onto the axis $T \geqslant /T_\zeta/$. But the expression for T_ζ may be represented in the following form:

$$T_\zeta = \sum_{i=1}^{A} {}^1\!/_2 \tau_\zeta^i = {}^1\!/_2 (Z - N),$$

where Z is the number of protons, N the number of neutrons and A the total number of nucleons, A = N + Z. Thus T_ζ equals half of the neutron excess of the nucleus taken with a reverse sign. (In published literature the old Wigner signs are frequently used whereby $\frac{1}{2}\tau_\zeta$ of a neutron equals ½. When this designation is used, the signs of the other formulas are also changed).

Hence if the isotopic spin of any state is T, such a state can be brought about only in systems where $/Z - N/ \leqslant 2T$. For example, states with T = 0 can be brought about only when Z = N, and states with T = 1 with Z = N or Z $\pm$ N.

As pointed out earlier, the introduction of the isotopic spins makes it possible to investigate a system of nucleons by the same methods used for a system of identical particles with a spin ½. In particular, it is fairly easy to generalize the methods of constructing a wave function of an electron system in the event of systems possessing an isotopic spin. We shall not go into that, however, inasmuch as we are not interested in the concrete calculation of the numerical level characteristics which call for an exact type of wave functions. A vector model would be sufficient for the purpose of this review.

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* Quantum mechanics shows that the law of quantum vector addition is a simple corollary of the commutation rules.

4. The Law of Isotopic Spin Conservation

Let us examine a system of Z protons and N neutrons in some stationary state, and assume that the hypothesis of the charge independence of nuclear forces is strictly fulfilled. As was already pointed out earlier, this assumption means that the properties of the system remain invariant when any neutron is replaced by a proton or vice versa. Let us replace one neutron by a proton. This will produce a new system but its Hamiltonian will be exactly equal to the old Hamiltonian. The stationary states of both systems must therefore be found from the solution of the same Schroedinger equation

$$H\psi = E\psi,$$

and the only difference here is made by the Pauli principle, according to which some of the states possible in one system are impossible in another. But the states that can be brought about in either system will obviously possess absolutely identical properties (they will have the same energy, momentum, parity, etc.).

To understand what all this means from the point of view of the isotopic spin, we shall point out the following. When one neutron is replaced by a proton, the projection of the isotopic spin system changes by the following unit: $T_\zeta \rightarrow T_\zeta + 1$ ($T_\zeta \rightarrow T_\zeta - 1$, the proton is replaced by a neutron), that is, such a substitution corresponds to a certain rotation in the isotopic space. Consequently, the fact that this substitution leaves the Hamiltonian function of the system invariant may be considered as the invariance of the Hamiltonian function of the nucleon system in relation to the rotations in an isotopic space. Hence the corollary that the complete isotopic spin of the system must be conserved. The proof in this case is absolutely analogical to the proof of the law of conservation of the ordinary angular momentum*. We thus come to a very important law: within the assumption of a

* We have cited a conditional but clear inference from the law of conservation of the isotopic spin. A strict deduction would be as follows: the operators of the transformation of a neutron into a proton and a proton into a neutron τ_2 and τ_1, especially τ_ζ, commute with the Hamiltonian function within the assumption of the charge independence. Therefore, the operators τ_ξ, τ_η, T_ξ, T_η & T_ζ representing the linear combinations $\tau_1^{(i)}$, $\tau_2^{(i)}$, also commute with the Hamiltonian function of the system. It thus follows that T is conserved.

charge independence of nuclear forces, the isotopic spin of the nucleon system is a motion integral.

The corollary of this conservation law is that every stationary state of the nucleon system (in a quantum-mechanical sense) must possess a definite isotopic spin. This follows directly from the general theory which states that any two commutative operators must possess a common system of "Eigenfunctions". Applying this theorem to the case when commutative operators are operators of the isotopic spin T and Hamiltonian H of the system, we get the formulation of the above assertion.

Chapter II. ISOTOPIC SPIN SELECTION RULES

5. Introduction

The law of conservation of the isotopic spin leads to definite selection rules for various nuclear reactions. These may be divided into two major types:

a) a reaction participated in only by nucleons whose complete isotopic spin is known to be conserved. In this case we address ourselves to the law of conservation of the isotopic spin and find that the entire nuclear process must occur in such a way that in each of its stages the isotopic spin of the system is equal to the initial isotopic spin;

b) a reaction participated in not only by nucleons but also by other particles (β-particles, γ-quanta) whose radiation changes the isotopic spin of the system which makes the direct application of the law of conservation of the isotopic spin impossible. In this connection, it is found, however, that under certain correlations between the isotopic spins of the initial and final states of a nucleon system, the matrix elements, conforming to radiation or absorption, are converted to zero identically. The conditions in which this occurs depend on the concrete form of interaction and determine the isotopic spin selection rules in this case.

Let us examine these cases in greater detail.

6. Reactions Caused by Nucleons Alone

In the interaction of heavy particles (neutrons, protons, heavy water, etc.) with light nuclei, the Coulomb interaction may be disregarded in comparison with the specific nuclear forces. It may be assumed therefore that in this nuclear region the hypothesis of the charge independence of nuclear forces is fairly accurate. This produces a number of interesting corollaries:

a) as the isotopic spin of heavy water and α-particles equals zero, the initial and final states of a nucleus in reactions of the (dd), (dα), (αd) and ($\alpha\alpha$) type must have the same isotopic spin. Moreover, if the reaction passes through an intermediate nucleus, the latter can be formed only in states whose isotopic spin equals the isotopic spin of the initial nucleus. This type of reaction therefore is a method of determining the isotopic spins of various nuclear states.

In the most interesting case, when the initial nucleus has an isotopic spin equal to zero, as it occurs in the ground states of almost all the 2n type of light nuclei, for example, we find that in such reactions the formation of a residual nucleus in a $T \neq 0$ state is impossible. Moreover, only the levels of an intermediate nucleus with a $T = 0$ state can be manifested in these reactions. In the $O^{16}(d\alpha)N^{14}$ reaction, for example, it was found that, with all the available heavy water (D_2O) energies, the group of α-particles, corresponding to the excited state N^{14} with an energy of $E = 2.31$ Mev, was missing. As the fundamental state of nucleus O^{16} is $T = 0$, and no other selection rules (with respect to moment or parity) can explain this exclusion, we learn from the law of the conservation of the isotopic spin that the isotopic spin $T \geqslant 1$ should be attributed to level N^{14} with $E = 2.31$ Mev. Indeed, we know from independent sources (5) that the isotopic spin of this level is $T = 1$.

By analogy, if the initial state of a nucleus was $T = \frac{1}{2}$, the reactions (dd) etc., of an intermediate state will produce only levels with a $T = \frac{1}{2}$ state, and the residual nuclei can be formed only in a state of $T = \frac{1}{2}$.

b) If an intermediate nucleus formed in any reaction is in a state with a definite isotopic spin T, the decay of that state must occur in such a way that the vector sum of the isotopic spin of the particles after the decay is equal to T. Thus if an intermediate nucleus is in a state of $T = 1$, the departure of an α-particle or D_2O is possible only when the nucleus formed in this process is also in a state of $T = 1$. In the $N^{15}(p\alpha)C^{12}$ reaction, for example, there is no manifestation of an excited state of the intermediate nucleus O^{16} with an excitation energy of $E = 12.95$ Mev. But it is known, from the point of view energy, that in this reaction the C^{12} nucleus can be produced only in states of $T = 0$ (the first state of $T = 1$ of a C^{12} nucleus has a very high excitation energy of ~ 15 Mev.).

Comparing these data, it is easy to conclude that the isotopic spin of the O^{16} level with E = 12.95 Mev must be equal to 1 (higher T are impossible for reasons of energy).

7. Selection Rules for A β-Decay

The calculation of the probabilities of a β-decay involves the calculation of the matrix elements of the operators dependent on the isotopic nucleon spins. It appears that many properties of such matrix elements can be found in general form. In this connection, use is made of the fact that the isotopic spin operators possess the same formal properties as the operators of the usual angle momentum. In particular, they satisfy the same commutation rules as the vector components of an angular momentum. On the other hand, it is known that the commutation rules make it possible to obtain a number of general formulas for matrix elements from various combinations of vector components of the angular momentum. Since the formulas obtained in this connection are based only on the commutation rules, they are fully applicable also to analogical matrix elements of isotopic spin operators.

We shall first present a summary of these selection rules for certain operators occurring in the calculation of probabilities of β- and γ-ray transitions.

a) If operator F is invariant in rotations in isotopic space (this happens when F, for example, is not at all dependent on the isotopic nucleon spin), the matrix elements of F fulfill the following conditions between the states with definite isotopic spins T, T' and their projections T_ζ, T'_ζ:

$$(TT_\zeta|F|T'T'_\zeta) \neq 0, \quad \text{only if} \quad T = T';\ T_\zeta = T'_\zeta.$$

b) If operator P_ζ is transformed into a vector ζ-component in the course of rotation in isotopic space (a simple example of such an operator is a T_ζ-operator of ζ-projection of a nuclear isotopic spin), the following selection rules are in operation: $(TT_\zeta|P_\zeta|T'T'_\zeta) \neq 0$, only if $T - T' = \pm 1; T_\zeta = T'_\zeta$ or if $T = T';\ T_\zeta = T'_\zeta \neq 0$.

c) If operator P_1, rotating in isotopic space, is transformed into $P_1 = (T_\xi + iT_\eta)$, the following rules apply to the selection of matrix elements from P_1:

$(TT_\zeta|P_1|T'T'_\zeta) \neq 0$, only if $T - T' = 0, \pm 1; T_\zeta = T'_\zeta + 1.$

d) If operator P_2, rotating in isotopic space, is transformed into $P_2=(T_\xi - iT_\eta)$, then $(TT_\zeta|P_2|T'T'_\zeta)\neq 0$, only if $T-T'=0, \pm 1$; $T_\zeta = T'_\zeta - 1$.

We shall now proceed to the β-decay itself. The transition of a nucleon from one charge state to another occurs in a β-decay (in a β-decay the neutron changes to a proton, and in a β+-decay the proton changes to a neutron). In this connection, the operator circumscribing the β-decay process looks like the following:

$$\mathfrak{M}_1 = \sum_i B_i(x, y, z, s_z)\tau_1^i \ (\beta- \text{ decay})$$

$$\mathfrak{M}_2 = \sum_i B_i(x, y, z, s_z)\tau_2^i \ (\beta+ \text{ decay})$$

where τ_1 and τ_2 are, respectively, the operators of the neutron change to a proton and, conversely, of the proton change to a neutron, while operators B_i are not dependent on the isotopic spin coordinates (according to the standard designation of the β-decay theory, $B_i = 1$ with respect to the scalar and vector variants, $B_i=\sigma_i$ with respect to the tensor and pseudovector variant and $B_i=\beta_i\gamma_5$ with respect to the pseudoscalar variant). The summation includes all the nucleons of the nucleus.

The expressions for $\mathfrak{M}_1$ and $\mathfrak{M}_2$ show that they change respectively into $\tau_\xi \pm i\tau_\eta$ during the rotation in isotopic space. We can therefore conclude at once that a β-transition is possible only between states whose isotopic spins T and T' differ by not more than 1: $T - T' = 0 \pm 1$. Here $\Delta T = 0$ in the case of the Fermi selection rules, and $\Delta T = 0 \pm 1$ in the case of the Gamov-Teller selection rules. Actually, the Fermi matrix elements (the scalar and vector variants of the β-decay theory) are, in a nonrelativistic case, reduced to the matrix elements of operator $\sum_i \tau_1^{(i)} = T_\xi + iT_\eta$ (or $\sum_i \tau_2 = T_\xi - iT_\eta$ in case of a β+-decay) which commutes with T^2. In such transition, therefore, T is conserved.

In the case of Gamov-Teller matrix elements (tensor and pseudovector variants) reducible to the matrix elements of operator $\sum_i \sigma_i\tau_1^{(i)}$ (or $\sum_i \sigma_i\tau_2^{(i)}$), additional limitations do not occur, and the usual selection rules for operators of this type are fulfilled: $\Delta T = 0, \pm 1$.

8. Selection Rules for γ-Radiation[6]

The selection rules for γ-transitions can be found by methods similar to those applied to the selection rules for β-decay. The transition operator (the Hamiltonian factor of the nucleon interaction with an electromagnetic field) in this case looks like the following (we disregard the very weak interaction connected with neutron and proton magnetic moments):

$$H = \sum_i \frac{e}{c} \cdot \frac{1}{2}\left(1 + \tau_\zeta^{(i)}\right) \mathbf{v}_i \mathbf{A}(\mathbf{r}_i),$$

where V_i, r_i represent the velocity and coordinate of an i nucleon, A the vector potential of the electromagnetic field, and the summation includes all the nucleons. This formula, in which all nucleons are absolutely symmetric, automatically takes into account the fact that the neutron does not interact with the electromagnetic field because of the absence of a charge. This is achieved by the introduction of operation $\frac{1}{2}\left(1 + \tau_\zeta^{(i)}\right)$, which, when acting upon the wave function of the nucleon, is equal to zero or 1, depending on the charge state of the nucleon (zero in the case of a neutron, and 1 in the case of a proton). Operator H may be rewritten as the sum of two parts:

$$H = H_0 + H_1,$$

where

$$H_0 = \sum_i \frac{e}{2c} \mathbf{v}_i \mathbf{A}(r_i); \quad H_1 = \sum_i \frac{e}{2c} \mathbf{v}_i \mathbf{A}(r_i) \cdot \tau_\zeta^{(i)};$$

H_o is not dependent on the isotopic spins of nucleons, and is therefore a scalar element in the isotopic space. Hence the selection rules for γ-radiation connected with this part of the Hamiltonian interaction: $\Delta T = 0$. The second addend, H_1, is transformed in the course of rotation in isotopic space as a vector ζ-component. The following selection rules therefore apply to the radiation connected with this part of the interaction operator:

$$\begin{array}{lll} \Delta T = 0, \pm 1 & \text{at} & T_\zeta \neq 0, \\ \Delta T = \pm 1 & \text{at} & T_\zeta = 0, \end{array}$$

that is in the case of a nucleus with $T_\zeta = 0 (N = Z) H_1$, transitions can occur only between levels with isotopic spins. On the other hand, it appears that H cannot lead to electric dipole transitions (E1), as the form of this part of the Hamiltonian coincides with the Hamiltonian circumscribing a system of identical particles with an $\frac{e}{2}$ charge, and such a system, as is known, cannot emit electric dipole radiation. Following from this is an important rule: dipole transitions between levels with the same isotopic spins are impossible in nuclei with $N = Z$.

This deduction is fully confirmed by experiment. Indeed, it was found that in nuclei with $T_\zeta = 0$ (B^{10} and N^{14}, for example) the E1-transitions between levels with the same isotopic spins are forbidden, whereas in the neighboring nuclei with $T_\zeta \neq 0$(Be^{10}, C^{14}) no exclusion has been found with respect to T.

It should be pointed out, however, that the exclusions of E1-transitions in connection with the isotopic spin are not absolutely rigid. They lower the probability of transition by several orders but do not completely forbid it. There are two reasons for that. First, the calculation of spin interactions is conducive to the possibility of E1-transitions and, second, each nuclear state has an admixture of states with other isotopic spins, and the presence of admixtures may bring about an E1-transition.

All the deductions of this paragraph apply to an equal degree both to the radiation processes of the γ-quanta with the transition of the nucleus to a lower state, and the absorption processes of the γ-quanta with a subsequent nuclear breakup. The reactions of the latter type produce a number of peculiar characteristics. Let us examine type 4n nuclei, for example. In such nuclei the ground state has an isotopic spin $T = 0$, and the first state with $T = 1$ is found only in the energy range of ~12-15 Mev. This leads to the existence of a threshold for the capture of electric dipole quanta, and the intensive γ-quanta capture and the subsequent nuclear breakup are therefore possible only at energies greater than ~15 Mev.

In some cases the threshold is located still higher. In a $C^{12}(\gamma\alpha)Be^8$ reaction, for example, the first excited state of a C^{12} nucleus with $T = 1$ is found at an energy of E = 15.2 Mev, and at this energy level a nuclear breakup into an γ-particle and Be^8, which is found in the ground state, is possible. However, in the ground state of Be^8, T = 0, and such a breakup is therefore forbidden by the law of the isotopic spin conservation. The reaction accompanied by the departure of an γ-particle will be resolved only if the energy of the γ-quanta is adequate for the formation of Be^8 in an excited state with T = 1. This corresponds to the energy of γ-quanta E $\sim$ 26 Mev. Thus in this reaction the threshold energy for an El-capture reaches a magniture of $\sim$ 26 Mev.

Similar exclusions occur also when nuclei[7] of the N = Z + 1 and A = 4n + 3 type are irradiated by low energy γ-quanta; inasmuch as the isotopic spin of such nuclei in a ground state is T = ½, the nucleus may change to a state with T = ½ or 3/2 by absorping γ-quanta. Excited states with T = ½ may decay in two ways: by emitting a neutron or a tritium nucleus. Indeed, the nuclei forming in the process of disintegration may, according to the law of the isotopic spin conservation, have T = 0 or 1 (the isotopic spin of tritium is T = ½), so that there is no exclusion of the isotopic spin. But if the excited state of the nucleus is T = 3/2, the residual nucleus can have an isotopic spin equal to 1 (or 2) but cannot have a spin equal to zero. Its decay therefore can occur only when the nucleogenesis in a state of T = 1 is resolved by energy. This makes the departure of tritium impossible.

Indeed, the escape of a neutron leaves an odd-odd nucleus with N = Z, and the T = 1 state of such nuclei is very close to the ground state. The escape of a tritium nucleus, on the other hand, should leave an even-even nucleus with N = Z where all the low-energy states have T = 0 (the first state with T = 1 is approximately within the E $\sim$ 12-15 Mev range), and that means that there is not enough energy to form a nucleus in a T = 1 state. Therefore, when the excitation energy of an initial nucleus is not very high, only a neutron can escape from the T = 3/2 levels, while a neutron and tritium can escape from the T = ½ level. It is thus possible to determine the isotopic spins of even-odd nuclei of the $A = 4n + 3$ type in excited states. That is how it was found that the first excited state of Li^7 with $T = 3/2$ is at a $E = 9.3$ Mev level.

Chapter III. THE ACCURACY OF THE ISOTOPIC SPIN

In the previous chapters we completely disregarded the neutron and proton properties (mass, charge, magnetic moments) which actually make it possible to distinguish these particles. Actually the neutron and protons are not completely equivalent; in particular, the neutrons are not subjected to the effect of Coulomb forces, whereas in the interaction of two protons account should be taken also of the interaction of their charges, in addition to the nuclear forces. The result is that the Hamiltonian of the nucleon system is, generally speaking, not a charge independent, and the nuclear spin cannot therefore be considered as an accurate quantum number. But in light nuclei, where the Coulomb interaction is small as compared to the nuclear forces (we shall establish the smallness criteria later), the charge-independent members of the Hamiltonian may be considered as a small addition to the "unperturbed" Hamiltonian, which is charge independent, and the ordinary perturbation theory can be used to calculate them. In this approximation, the complete isotopic spin will no longer be conserved so that the states of the nucleon system will become a mixture of states with various isotopic spins, but as long as we can consider the charge-independent members of the Hamiltonian as merely a small addition, only one value of the isotopic spin (the unperturbed state) will play a dominant part in that mixture. The mixtures in this case are small, and the isotopic spin continues to retain its value as an approximate quantum number characterizing the various states of the nucleon system.

9. The Release of Charge-Independent Members

The exact Hamiltonian of a nucleus may be recorded as follows:

$$H = V_0 + \frac{1}{4}\sum_i P_i^2\left(\frac{1+\tau_\zeta^{(i)}}{m_p} + \frac{1-\tau_\zeta^{(i)}}{m_n}\right) + \\ + \sum_{i>k}\frac{e^2}{4r_{ik}}\left(1+\tau_\zeta^{(i)}\right)\left(1+\tau_\zeta^{(k)}\right),$$

where V_0 is the term describing the nuclear interaction of nucleons (assumed to be charge independent), and the second and third terms represent, respectively, nucleon kinetic energy and Coulomb interaction energy. This expression may be rewritten as indicated below by singling out the kinetic energy of the charge-independent part of the operator

$$H = V_0 + \sum_i \frac{P_i^2}{4}\left(\frac{1}{m_p} + \frac{1}{m_n}\right) +$$
$$+ \frac{m_n - m_p}{m_n} \sum_i \frac{P_i^2}{2m_p} \tau_\zeta^{(i)} + \sum_{i>k} \frac{e^2}{4r_{ik}} (1 + \tau_\zeta^{(i)})(1 + \tau_\zeta^{k}) =$$
$$= H_0 + \frac{m_n - m_p}{m_n} \sum_i \frac{P_i^2}{2m_p} \cdot \tau_\zeta^{(i)} +$$
$$+ \sum_{i>k} \frac{e^2}{4r_{ik}} (1 + \tau_\zeta^{(i)})(1 + \tau_\zeta^{(k)}) = H_0 + v_1 + v_2,$$

where H_o is the charge-independent part of the Hamiltonian, and v_1 and v_2 are the charge-independent parts of the kinetic energy operator and the Coulomb energy operator, respectively. In this expression H_o is the "unperturbed" Hamiltonian, and v_1 and v_2 are the additions whereby the isotopic spin is no longer a quantum number. This means that the wave function of the nucleus is the sum of wave functions pertaining to various values of the isotopic spin. We shall assume that some single value of the isotopic spin plays a major part, and that all the other functions represent a small "admixture." This is a legitimate assumption as we are interested in the accuracy of the isotopic spin in the region of light nuclei where the charge-independent part is a small perturbation.

Let us examine a system of states with a preset moment J, parity P and isotopic spin T. We will designate the wave functions of these states as

$$\psi_m \, (m = 0, 1, \ldots).$$

These states are distinguishable by their isotopic spin and, possibly, by some other quantum numbers*, and represent the "eigenfunctions" of the unperturbed operator H_o. Let the state be characterized primarily by wave function ψ_o. The accurate wave function of the nucleus will then look like this

$$\psi = \psi_0 + \sum \alpha_{0m} \psi_m.$$

---

* In a system consisting of many parts, the prescribed J, P, T do not by themselves determine the state. Generally speaking, a further classification depends on the concrete properties of the nuclear forces.

The coefficients α_{om} are found by the known formula of the perturbation theory. We are interested only in the square of their moduli:

$$|\alpha_{0m}|^2 = \left| \frac{(\psi_0|v|\psi_m)}{(E_0 - E_m)} \right|^2,$$

where the matrix perturbation element v = $v_1 + v_2$ is the numerator calculated by the use of the wave functions of the unperturbed operator H_o.

The degree of "purity" of the state is usually characterized by the "portion of the admixture" (Radicati[8,9])

$$\xi = \sum_{m \neq 0} |\alpha_{0m}|^2.$$

This evaluation calls for the knowledge of ψ_m functions. It is possible, however, to make a simple estimate of the sum by substituting some average difference (ΔE) for the energy difference. Then by replacing the summation of m $\neq$ 0 by that of all m, we get

$$\xi = \sum_{m \neq 0} \left| \frac{(\psi_0|v|\psi_m)}{(E_m - E_0)} \right|^2 < \frac{1}{(\Delta E)^2} \sum_m (\psi_0|v|\psi_m)(\psi_0|v|\psi_m)^* =$$

$$= \frac{1}{(\Delta E)^2} \sum_m (\psi_0|v|\psi_m)(\psi_m|v|\psi_0) = \frac{1}{(\Delta E)^2} (\psi_0|v^2|\psi_0).$$

In these transformations we used the Hermitian character of the v matrix and the rules of matrix multiplication.

The matrix element $(\psi_0|v^2|\psi_0)$ can be easily evaluated from the experimental data, as this is simply an average value of the square of operator v = $v_1 + v_2$. It is not difficult to see that the member v_1 may be disregarded in comparison with v_2. Actually, by defining the operators v_1 and v_2 it is easy to get the following estimates of their average values:

$$\bar{v}_1 = \frac{m_n - m_p}{m_n} \cdot \overline{\sum_i \frac{P_i^2}{2m_p} \tau_\zeta^{(i)}} \approx \frac{m_n - m_p}{m_n} \cdot \varepsilon \cdot T_\zeta,$$

where ε is the nucleon kinetic energy in the nucleus ($\varepsilon \sim$ 8 Mev). Hence the order of magnitude $\bar{v}_1 \sim$ 0.01 Mev, whereas

$$\bar{v}_2 = \sum_{i>k} \frac{e^2}{4r_{ik}} (1 + \tau_\zeta^{(i)})(1 + \tau_\zeta^{(k)}) \approx \frac{Z(Z-1)}{2} \cdot 0{,}5 \text{ Mev}$$

where Z is the nuclear charge, and 0.5 Mev the average energy of the Coulomb interaction between two protons in a nucleus. We thus arrive at the conclusion that the calculation of ξ should take into account only the Coulomb energy of the protons, as the effect produced by the difference in the proton and neutron mass is very small. We get the following evaluation:

$$\xi \leqslant \frac{\bar{v}_2^2}{(\Delta E)^2}.$$

The definition of ΔE requires a knowledge of the position of many levels. For orientation purposes, we can replace ΔE by the distance to the nearest level with a different isotopic spin but with the same J and P. In this way we will find the upper boundary of ξ.

Such an evaluation leads us to the value $\xi \approx 10^{-3}$-10^{-4} for Be and $\xi \approx 0.1$—0.5 for O^{16}. These values are obviously too high. An analysis of the experimental data on the violation of the selection rules produces lower values[8-12]. The idea of the isotopic spin of ground states apparently makes sense until $Z \sim 20$. In the case of excited states, it ceases to be a quantum number considerably earlier.

The general conclusion that highly excited states do not have a definite isotopic spin is confirmed by the experimental data[13]. Thus an analysis of the data on the reactions of $N^{15}(p\alpha)C^{12}$ (ground state) and $N^{15}(p\gamma)O^{16}$ (ground state) established that the excited state of O^{16} with an energy of 13.09 Mev apparently does not have a definite isotopic spin but is a mixture of states with T = 0 and T = 1 (this conclusion is based on the fact that at this level the probability of disintegration into $C^{12} + \alpha$ and $O^{16} + \gamma$ is about equal, whereas, according to the selection rules of the isotopic spin, the first disintegration is possible only if O^{16} has T = 0, and the second method of disintegration requires T = 1). Another level is currently known which, apparently, does not have a definite isotopic spin, and that is the excited state of the B^{10} nucleus with an energy of 7.48 Mev. Just as in the first case, such a conclusion is based on the high probability of this state decaying both under the $B^{10*} \to \alpha + Li^6$ and $B^{10*} \to B^{10} + \gamma$ scheme.

Chapter IV. SIMILAR LEVELS OF LIGHT NUCLEI

10. Similar Levels*

As has already been mentioned more than once, the Coulomb interaction between protons in light nuclei with $Z \leqslant 15 - 25$ is small as compared to the specific nuclear forces. Hence the corollary (see chapter I, Section 5) that the substitution of a neutron for a proton in a nucleus, and vice versa, will produce a new nucleus whose Hamiltonian is very little different from that of the first nucleus; the only essential difference is that some of the states possible in one of these nuclei will be excluded from another by the Pauli principle. But the states that are possible in both of the nuclei will possess the same properties, and it is these states (similar states) that will have the same momentum, parity, isotopic spin and internal structure; the energy difference between two such states in one nucleus will almost exactly coincide with the energy difference between corresponding states in another nucleus, etc. The difference between these two nuclei will be due to the difference in the Coulomb energy. In the majority of cases, however, the result is that all the levels of one nucleus simply shift in relation to the corresponding levels of another nucleus, and the energy difference between the corresponding levels changes by an insignificant amount (some exceptions to this rule will be dealt with later). We should point out here that the Coulomb shift facilitates a simple calculation of the electric interaction energy of the protons in the nucleus. To this end, similar levels should be found, and their energies compared, in two nuclei in which a proton was substituted for a neutron. The difference between these energies, making allowances for the difference between the neutron and proton mass, will reveal the Coulomb energy per one proton. This is apparently the most direct and simple method of determining the Coulomb energy of light nuclei.

* See also the work of B. S. Dzhelepov[14], and the recently published review by B. S. Dzhelepov[26].

From the point of view of the properties resulting from the hypothesis of charge independence, the light nuclei are divided into two basic groups: the group of type 2n nuclei, even-even and odd-odd nuclei, and the group of type 2n + 1 nuclei, odd-even nuclei. The first group is characterized by the fact that inasmuch as the nuclei of this group consist of an even number of nucleons, the various states of these nuclei can have only integral values of the isotopic spin: 0, 1, 2, ... The basic characteristic of the energy levels of this type of nuclei is that the energy states with $T = 0$ are more convenient than the states with $T = 1$, and the latter, in turn, is more convenient than a state with $T = 2$, etc. This brings up the concept of nuclear triads, that is, triplets of isobaric nuclei with various neutron-proton ratios (for example Be^{10}, B^{10} and C^{10}). Two nuclei of such a triad have $T_\zeta = +1$ and $T_\zeta = -1$ (C^{10} and B^{10} respectively), and the third has $T_\zeta = 0(B^{10})$. In this connection, the first two nuclei can be only in states of $T = 1, 2, \ldots$, and the third can be also in a state of $T = 0$. Thus the $T = 0$ states occur nuclei with $T_\zeta = 0$, and have their analogues in the other nuclei of the triad; but the $T = 1, 2$ states may occur in all members of the triad, and for each such state in one of the nuclei there is a corresponding similar state in the other members of the triad, and the moments, parities, relative location and other characteristics of the similar levels are the same in all members of the triad.

The nuclei of the 2n + 1 type consist of an odd number of nucleons; their states can possess only half-integral values of the isotopic spin $T = ½, 3/2, \ldots$ The states with $T = ½$ are found to be more convenient energywise than those with $T = 3/2$, and, as a result, all the stable nuclei of this type with $/T_\zeta/ = ½$ are primarily in a state of $T = \pm½$. Thus the nuclei in this case are also grouped into pairs of isobaric nuclei with isotopic spin projections $T_\zeta = ½$ and $T_\zeta = -½$ (so-called mirror nuclei).

The triads of isobaric even nuclei and the pairs of isobaric uneven nuclei with $T_\zeta = \pm½$ are referred to in literature by the common name of charge multiplets. We should point to still another term found in literature, and that is super multiplets. This term is used in the classification of nuclear states on the assumption of a specific L-S-coupling. Here the symmetry of the space portion of the nuclear wave function is determined by three numbers: the ordinary and isotopic nuclear spins S and T and the additional quantum number Y which characterizes the symmetry of the wave function product of ordinary and isotopic spins. The states of two isobaric nuclei having the same S, T and Y values are referred to as belonging to one super multiplet.

11. Methods of Determining the Isotopic Spin of Nuclear States

The determination of the isotopic spins of light nuclei in various states is usually guided by the following considerations:

a) In many cases the use of the selection rules for the isotopic spin makes it possible to determine the isotopic spin of a particular state of a nucleus, if the isotopic spin of the initial or residual particles is known.

b) Energy considerations are frequently helpful in determining the isotopic spin of certain nuclei. Let us examine an α-particle, for example. Since it consists of two neutrons and two protons ($T_\zeta = 0$), the ground state can be only T = 0, 1, 2. It is easy to see, however, that the ground state of an α-particle is T = 0 for otherwise it would facilitate the existence of a stable isotope H^4 with a binding energy only slightly different from the binding energy of an α-particle (a slight Coulomb shift). Experience shows, however, that such a state H^4 does not exist. This is a decisive argument in favor of the fact that the isotopic spin of an α-particle in a ground state is equal to zero.

The same method is used to prove that the isotopic spin of mirror nuclei H^3 and He^3 is equal to ½. Actually, if their isotopic spin were equal to 3/2, there would exist stable nuclei consisting of three neutrons or three protons, which has not been observed. Finally, the very same method can be used to conclude, from the lack of a stable state of a two-neutron system, that the isotopic spin of D_2O is equal to zero.

Similar considerations are frequently helpful in determining the isotopic spins of heavier nuclei in various states. In particular, this type of reasoning is a basic argument in favor of the fact that the ground states of almost all even-even and odd-odd light nuclei have T = 0. To illustrate, we shall take a Be^8 nucleus. It is known that the binding energy of a $Be^8(T_\zeta = 0)$ nucleus is $\sim$ 16 Mev greater than that of an $Li^8(T_\zeta = -1)$ nucleus which differs from Be^8 only by the substitution of one neutron for a proton, and therefore can be only in T = 1 states. On the other hand, it is clear that the Coulomb energy must be higher in Be^8 which has one more proton. It is easy to conclude therefore that the energy of the first state of Be^8, which also may occur in Li^8 (a T = 1 state), is at any rate not less than 16 Mev. Hence the corollary that the ground and all the excited states of a Be^8 nucleus with an excitation energy of less than $\sim$16 Mev should have T = 0.

c) The isotopic spin of certain states can be determined from the structure of a given state, that is from the known state of nucleons in a given state of the nucleus. It is known, for example, that the wave function of the relative nucleon movement in D_2O (heavy water) is a superposition of $^3S_1$ and $^3D_1$ states which are symmetrical in relation to the permutation of nucleon space coordinates and spins. Inasmuch as the entire wave function must be antisymmetric, the isotopic spin of these states is equal to zero. It thus follows from the known wave function of D_2O, in accordance with the previously obtained result, that D_2O in a ground state has T = 0.

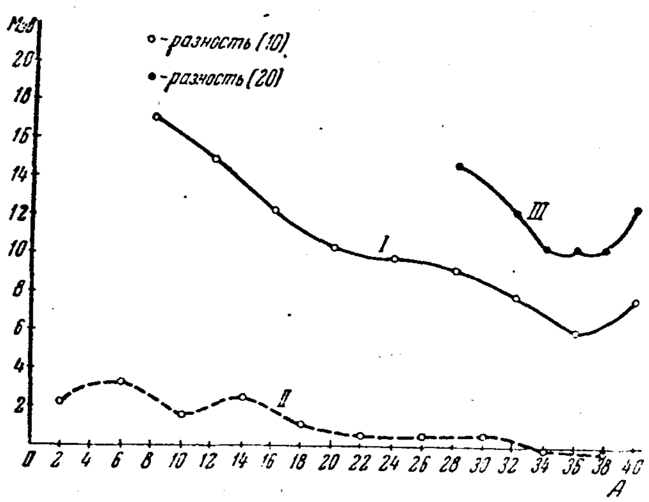
It is known that in addition to the ground triplet state, the O_2D has a virtual singlet state ^1S which manifests itself in neutron-proton scattering. The ^1S state has T = 0, and an analogical virtual state is therefore found in a two-neutron or two-proton system which is borne out by experiment.

Structural considerations play an important part in determining the isotopic spins of various nuclear states as, according to the shell model, the nucleons in a nucleus are in states with preset orbital angular momentum. In this case, it is found possible to calculate the moments and isotopic spins of all the states of such a system, and a knowledge of the moment of a particular nuclear state, therefore, may occasionally make it possible to draw a conclusion about the isotopic spin of that state (when there is a single-valued relation between the isotopic spin and moment of the state).

12. Certain Characteristics of the Level Arrangement

The data on the light nuclear levels cited in the second part make it possible to establish a number of regularities.

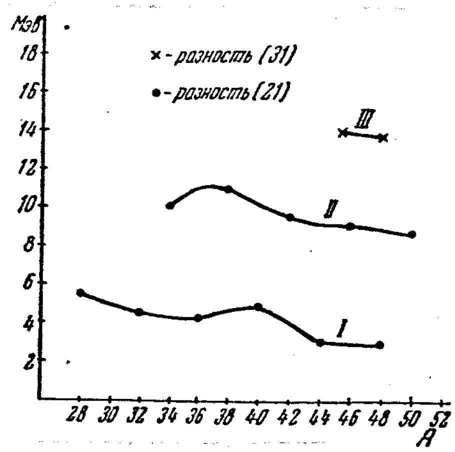
Let us plot on a chart the energy differences between the lower levels with T = 0, 1, 2, 3 of nuclei with an even mass number A. The striking characteristics of such charts are that the points corresponding to the differences between the lower levels with T = 1 and T = 0 (a difference of (10)), as seen from Fig. 1, are plotted on two smooth curves, one for type 4n nuclei (curve I), and the other for 4n + 2 nuclei (curve II). The first curve changes from 17 Mev in Be^8 to 6 Mev in A^{36} in the $6 < A < 40$ domain, whereas the second curve never rises above 3.6 Mev (Li^6). Similarly, the points corresponding to the energy difference between the first levels with T = 2 and T = 1 (a difference of (21)) are plotted on two smooth curves, one for type 4n nuclei and the other for type 4n + 2 nuclei (Fig. 2).



разность = difference

Мэв = Mev

Fig. 1



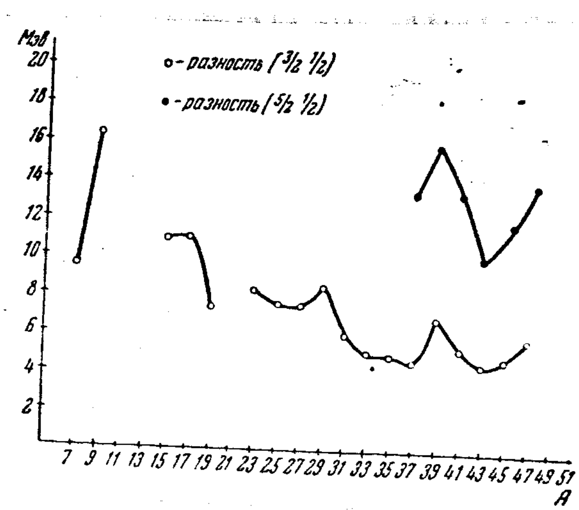
разность = difference

Мэв = Mev

Fig. 2

The first curve (4n) never rises above 5.5 Mev (Si^{28}), whereas the second curve (4n + 2) of the same range is entirely included in the narrow band of 8.5-11.0 Mev. In contrast to the differences (10) and (21), the energy differences between the first levels with T = 2 and T = 0 as well as T = 3 and T = 1, whose isotopic spins differ by 2 (differences of (20) and (31)), are smooth functions of a mass number. In this case, the 4n and 4n + 2 nuclei are plotted on the same curve (curve III in Fig. 1 and 2).

If we plot a similar chart (Fig. 3) for nuclei with an odd A, we will find that all the points (3/2, ½) and (5/2, 3/2) are arranged on relatively smooth curves.



разность = difference

Мэв = Mev

Fig. 3

Considerations resulting from the shell theory, reactions, differences in the binding energy of isobaric nuclei etc. are used in determining the isotopic spins of nuclei with $Z < 10$. In the field of nuclei with $10 < Z \leqslant 25$, it is impossible to use the data on nuclear reactions, as the law of the isotopic spin conservation is apparently no longer operative, or considerations of the shell theory, inasmuch as it is still not very clear how the various shells are filled in this domain. The chief method of determining the isotopic spins in this case is a comparison of the isobaric nuclear levels. This brings up the question of the authenticity of the identification of states by the isotopic spin of these nuclei; moreover, it is necessary to make certain that the isotopic spin still retains its significance with respect to these nuclei. The following may be said in this connection. In the field of nuclei with $Z < 10$, where the validity of introducing an isotopic spin as a quantum number is fully borne out by all experimental materials, there is a conspicuous relationship between the difference (10), etc., and the mass number A. The fact that these regularities are still manifested in $Z > 10$ leads to the conclusion that the isotopic spin is a characteristic state of such relatively heavy nuclei.

In a number of cases the above-established regularities justify the conclusion about the stability or instability of a particular isotope, and the indication of its approximate binding energy. Isotope Al^{30} with $T_{\zeta} = -2$, for example, is still unknown. Extrapolating curve III onto Fig. 2, we find that the first state with $T = 2$, which should be a ground state of Al^{30}, is approximately 15 Mev higher than the first state with $T = 1$ (ground state of Si^{30}). Taking into account the Coulomb energy differences of Si^{30} and Al^{30} (which is 5-6 Mev), we come to the conclusion that the binding energy of Al^{30} should be ~ 9 Mev less than Si^{30}. It may be concluded, in view of the possible disintegration methods of this isotope, that the only possible method of Al^{30} decay is the β-decay according to the $Al^{30} \beta- \rightarrow Si^{30}$ scheme. The existence of the following heretofore unknown isotopes can be shown by similar methods: Na^{26} with a binding energy ~ 12 Mev less than in Mg^{26}, Ne^{24} with a binding energy $\sim$ 4 Mev less than in Na^{24}, Cl^{40} with a binding energy $\sim$ 5-6 Mev less than in Ca^{40}, etc. Thus the binding energy of K^{44} should be $\sim$ 5 Mev less than in Ca^{44}, and the binding energy of A^{42} $\sim$ 1-2 Mev less than the binding energy of K^{42}. It would be very interesting to verify these predictions.

PART II. THE ENERGY-LEVELS OF LIGHT NUCLEI

This part contains a brief summary of the energy-levels of light nuclei for $A \leqslant 50$. The purpose of the figures in this review is to illustrate the relative arrangement of the isobaric levels and provide an idea of the isotopic spin distribution in nuclei. The review does not exhaust all the known material*, and it is probably not free of certain arbitrary assumptions in the matter of identification.

The energy levels are arranged in such a way that the similar states of neighboring nuclei are combined. To this end, the difference in the isobaric mass, known from the tables, was adjusted for Coulomb energy and the difference in the proton and neutron mass. The resulting magnitude thus determines the distance between the ground states in the energy-level schemes. As the Coulomb energy is indeterminate, such an arrangement is not very accurate, and an error of several hundred Kev [kilo-electron-volts] is not improbable. It should be remembered that, in view of the above-said, the level arrangement of the neighboring nuclei does determine the energy of the $\beta_{\pm}$-spectra.

The diagrams show that the experimental data are distributed very unevenly among the various nuclei, and that a more detailed analysis of such nuclei calls for further experiments.

He^4, Li^4 and H^4

He^4 has $T_\zeta = 0$ which makes the T = 0, 1, 2 states possible in it, and in Li^4 and H^4 T_ζ is equal, respectively, to +1 and -1; consequently, states with T = 0 cannot occur in them. H^4 has, in addition to a ground state with T = 0, two more excited states with energies of ~22.5 and ~23 Mev.[15] The isotopic spins of these states are, apparently, equal to zero as they are not manifested in a $T(p\gamma)H^4$ reaction. It follows that the possible states of Li^4 and H^4 should have a still

* With few exceptions, the data on energy levels were borrowed from the two reviews mentioned above[3,4].

higher energy, and as the dissociation of T + p or H^3 + p becomes possible at > 21 Mev, stable states cannot exist in H^4 or Li^4.

Li^5 and He^5

Li^5 and He^5 are unstable mirror nuclei with a similar system of levels. The three currently known level possess an isotopic spin T = ½, as the first two levels of He^5 are manifested in the H^4 + n reaction (it should be recalled that the isotopic spin of an α-particle is equal to zero, and that of a nucleon to ½), and the level with E = 16.8 Mev manifests itself in a d + T reaction.

Li^6, He^6 and Be^6

States with T = 0, 1, ..., are possible in $Li^6(T_\zeta = 0)$, but T = 0 states cannot materialize in $Be^6(T_\zeta = 1)$ and $He^6(T_\zeta = -1)$. Thus He^6 can have all the Li^6 levels except those with T = 0. Hence the Li^6 level, corresponding to the ground state of He^6, should have T = 1 (the ground state of He^6 should be T = 1 for, if it were equal to 2, there would exist a superheavy isotope of hydrogen He^6 which does not exist in nature). To find that level, we will point out that if the Coulomb energy and the difference between the neutron and proton mass were left out of account, the energy difference between the ground states of He^6 and Li^6 should be exactly equal to the energy of the first level with T = 1. The average energy of the Coulomb interaction between two protons in light nuclei is 0.4-0.5 Mev, and the difference between the neutron and proton mass is 0.78 Mev. Comparing these figures, it is easy to conclude that the ground state of He^6 corresponds to the Li^6 level with E = 3.58 Mev, to which we must therefore assign T = 1. The immediate inference is that the ground and first excited states of Li^6 have T = 0.

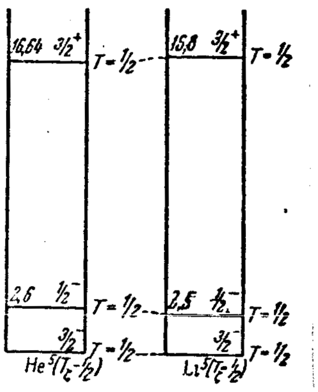


Fig. 4.

The relative arrangement of the isobaric nuclear levels in Fig. 4 is different from the experimentally observed arrangement in connection with the difference between the Coulomb energies of these nuclei, and the correction that takes into account the difference between the neutron and proton masses. In such a combination, the similar levels (indicated in the Figures by dotted lines) of isobaric nuclei coincide.

This conclusion is confirmed by the results[16] of the $Be^9(p\alpha)Li^9$ reaction. Namely, the compound state B^{10} with an energy of E = 8.89 Mev produced by this reaction decays according to the $\alpha + Li^6$ scheme*; this produces Li^6 only in a state with an energy of E = 3.58 Mev, but not a ground of first excited state of Li^6. Hence it appears that the isotopic spin of the second excited state of $Li^6(E = 3.58)T \neq 0$.

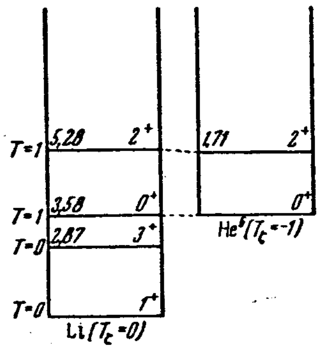


Fig. 5.

The nucleus $Be^6(T_\zeta = 1)$ should have been analogous to He^6. However, in view of the relatively high Coulomb energy (approximately 1.5 Mev higher than in Li^6), this nucleus is unstable in regard to a split into He^4 + 2p, and therefore does not occur in nature.

Li^7 and Be^7

Li^7 and Be^7 are a pair of mirror nuclei with an absolutely similar system of levels. Hereafter we will therefore speak only of Li^7 levels. The ground and first two excited states of Li^7 are manifested in the reaction $Li^6 + d \rightarrow Li^7 + p$ which justifies the conclusion that they all have T = ½. The level with E = 7.5 Mev is manifested in the reaction $Be^9(d\alpha)Li^7$ along with the first three levels of Li^7 which have T = ½. This level should therefore also be assigned T = ½. An analysis of the (γn) and (γT) reactions shows (see chapter II) that the levels with E = 9.6 and 17.5 Mev have T = 3/2, and those with E = 14, 12.4 and 10.8 Mev should be assigned T = ½. What is still unclear is the isotopic spin of the level with E = 6.6 Mev. Inasmuch as this level manifests itself in reaction (pp') but not in reaction $Be^9(d\alpha)Li^7$, its isotopic spin may be expected to be equal to 3/2.

This conclusion, however, should be approached with caution as it has not yet been tested in other reactions (in $Li^7(\alpha\alpha')Li^7$, for example*).

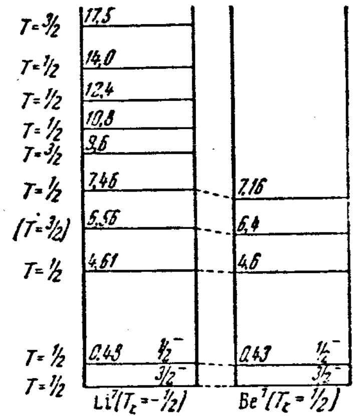


Fig. 6.

B^8, Li^8 and Be^8*

As mentioned in the preceding paragraph, it is clear from considerations of energy that all the Be^8 levels with $E < 16$ Mev have $T = 0$. The exact energy of the first level with $T = 1$ is still not clear but it may be expected to be $E = 17$ Mev. This is deduced from the reaction $C^{12}(\gamma\alpha)Be^8$ in which the selection rules of the isotopic spin make the formation of Be^8 levels with $T = 1$ highly probable. A study of that reaction showed that Be^8 is formed primarily in a state with $E = 16.7$ Mev[18], and this accounts for the above-cited assertion.

The ground and first excited states of Li^8 have $T = 1$. This follows from the fact that both of these states manifest themselves in the reaction $Li^7(dp)Li^8$.

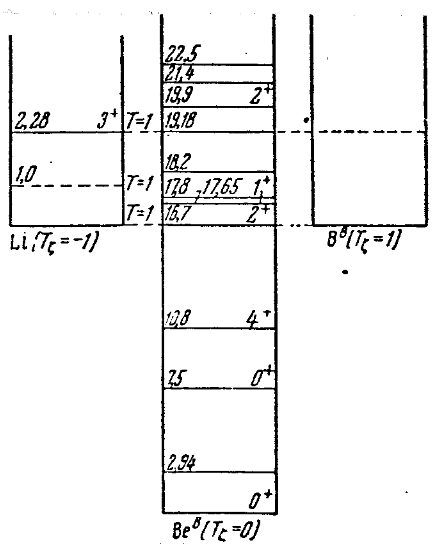


Fig. 7.

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* The diagram of the Be^8 levels is based on the work of Bonner and Cook[17].

In view of its relatively high Coulomb energy, the B^8, third member of the triad, has a very short lifetime, and has practically not been studied; however, since the binding energy of Li^8 and B^8, minus the Coulomb energy and the difference between the neutron and proton mass, is almost exactly the same, it follows that B^8 is an analogue of Li^8, and has T = 1 in its ground and first excited states.

B^9 and Be^9.

B^9 and Be^9 are mirror nuclei, and this is attested to by the fact that the binding energies of these nuclei, minus the Coulomb energy and the difference between the neutron and proton mass, are almost exactly the same. This fact is very important for, although the B^9 nucleus is unstable in regard to a decay into Be + p and was therefore inadequately studied (a small Coulomb shift is found to be adequate for this purpose, as the ground state of Be^9 is only 1.6 Mev lower than threshold of disintegration into Be^8 + n), its excited states can manifest themselves in certain reactions; since B^9 and Be^9 are mirror nuclei, we may conclude that all their lowlying levels are similar and, consequently, it would be enough to study only the excited states of Be^9.

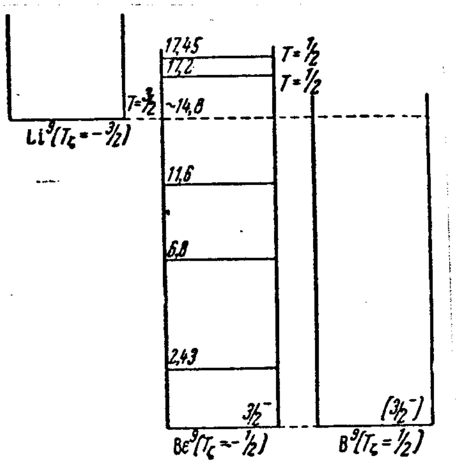

Fig. 8.

The binding energy of the isobaric nucleus Li^9 $(T_\zeta = -3/2)$ is 14.1 Mev lower than in Be^9(T = -½). Hence we immediately conclude that all Be^9 states with E $<$ 14 Mev have T = ½. This accords with all the reactions observable in this energy region. This also justifies the conclusion that there must be a level with

$T = 3/2$ in Be^9, somewhere in the area of $E \sim 15$ Mev (this figure was obtained from the difference between the binding energy of Be^9 and Li^9 which should be corrected for the Coulomb interaction of a superfluous proton in Be^9 and for the difference between the neutron and proton mass); that level, however, has not yet been found as there are no suitable reactions in that energy area.

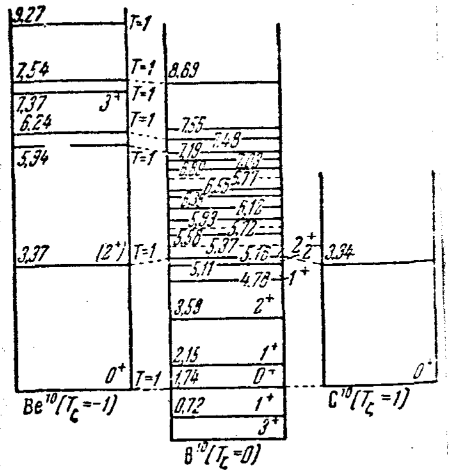

Fig. 9.

The following levels of Be^9 with $E = 17.2$ and 17.45 Mev apparently have $T = ½$ as they manifest themselves in $Li^7 + d$ reactions.

Be^{10}, B^{10}, C^{10}

The $B^{10}(T_\zeta = 0)$ nucleus may be found in states with $T = 0, 1, 2, \ldots$, while $B^{10}(T_\zeta = -1)$ and $C^{10}(T_\zeta = 1)$ cannot occur in states with $T = 0$. Comparing the ground state energy of the B^{10} and Be^{10} nuclei, it is easy to deduce that the ground state of B^{10} has $T = 0$. Hence the corollary that the ground states of the B^{10} nucleus with $E = 0.72$, 2.15 and 3.58 Mev also have $T = 0$ as they all manifest themselves in reaction $B^{10}(dd')B^{10*}$. The state with $E = 1.74$ is not manifested in that reaction, and it should be assigned $T = 1$, that is, the state which should occur as a ground state in B^{10} and C^{10}. This deduction is fully confirmed by considerations of energy as the binding energy of B^{10} (if the Coulomb energy and the difference between the neutron and proton mass are taken into account) exceeds the binding energy of B^{10} by exactly ~ 1.7 Mev.

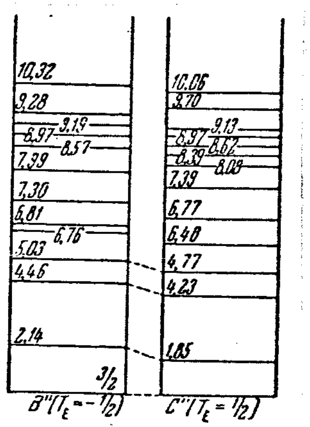

Fig. 10

Applying the law of the isotopic spin conservation to the various reactions occurring in the excited states of Be^{10} with an excitation energy of $<$ 10 Mev, it is easy to find that all these states of Be^{10} have T = 1. Corresponding states should be found also in B^{10}. It has now been almost conclusively established[19,20] that the level of Be^{10} with E = 3.37 Mev corresponds to that of B^{10} with E = 5.16 Mev, the level with E = 5.94 Mev to that with E = 7.19, the level with E = 6.24 to that with E = 7.48, and the level with E = 7.54 Mev to that with E = 8.89 Mev. Used in this comparison were the results of various reactions in connection with the selection rules of the isotopic spin. For example, the B^{10} level with E = 8.89 Mev is assigned T = 1 as, first, it is from this level that the decay to Li^6 in a state of E = 3.58 Mev (in this state T = 1) and an α-particle occurs, and, second, the intensive γ-transition to a ground state of B^{10} (T = 0) starts from this level; and this could not have occurred if the state of B^{10} with E = 8.80 Mev had T = 0.

The C^{10} nucleus has been studied much less than the Be^{10} but it is clear from considerations of energy, that it is an analogue of B^{10} (the difference in the binding energies of B^{10} and C^{10}, taking into consideration the Coulomb energy and the difference between the neutron and proton masses, is, as in the case of Be^{10}, equal to $\sim$ 1.7 Mev). The level arrangement in C^{10} must therefore be exactly the same as in Be^{10}.

B^{11} and C^{11}.

These are mirror nuclei with a similar system of energy levels and $T_\zeta = \pm\frac{1}{2}$. Leaving the Coulomb energy and the difference between the neutron and proton masses out of account, the binding energies of the C^{11} and B^{11} nuclei are equal. It follows from reaction $B^{10}(dp)B^{11}$ that all the states of B^{11} (and therefore also C^{11}) with an excitation energy of $E < 9$ Mev have $T = \frac{1}{2}$. This explains the absence of any stable N^{11} and Be^{11} isotopes, as these nuclei have $/T_\zeta/ = 3/2$ which means that the binding energy of these nuclei should be at least ~ 9 Mev less than the binding energy of B^{11} and C^{11}. With such low binding energy the Be^{11} and C^{11} nuclei cannot exist, and they decay to Be^{10} + n and C^{10} + p, respectively.

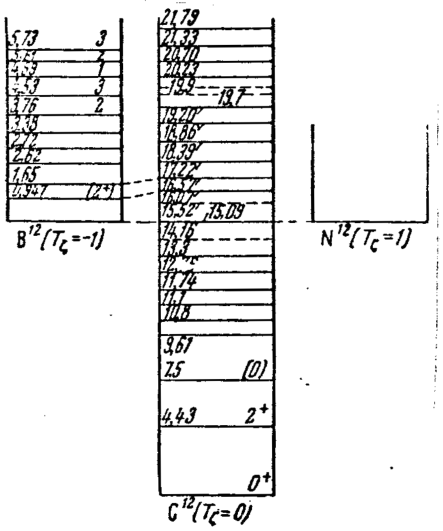

Fig. 11.

B^{12}, C^{12} and N^{12}.

The binding energy of the $C^{12}(T_\zeta = 0)$ nucleus, minus the Coulomb energy and the difference between the neutron and proton masses, is approximately 15 Mev greater than the binding energy of the $B^{12}(T_\zeta = -1)$ and $N^{12}(T_\zeta = 1)$ nuclei. It follows that all the excited states of C^{12} with $E < 15$ Mev must have $T = 0$. This deduction is conformed by a large number of reactions: for example, the reaction of the radiative capture of $C^{12} + \gamma$ practically does not occur at a γ-quantum energy of less than 15 Mev (it should be recalled that since in C^{12} $T_\zeta = 0$, electron-transitions can occur only between levels with different isotopic spins). The position of the first level of C^{12} with $T = 1$ is still not very clear, but there are indications[21] that it is a level with E = 15.9 Mev*. If this deduction is correct, the level of C^{12} with E = 15.09 Mev is analogical to the ground state of B^{12} and N^{12}. Data are now available[23] on one more level of C^{12}, namely a level with E = 16.07 Mev. The escape of an α-particle is possible from this level, which has a moment of 2 +, but the corresponding width is very small ($\Gamma_{\gamma} \sim 5$ kilo-electron volts, whereas a width of ~ 1 Mev should be expected at such energies). This is treated as an indication that the isotopic spin of this state of C^{12} is equal to 1 (the fact that the departure of an α-particle is still possible is attributed to the admixture of states with T = 0 in this level of C^{12}). If such an identification is correct, the C^{12} level with E = 16.07 Mev is an analogue of the first excited state of the B^{12} and N^{12} nuclei.

The B^{12} and N^{12} nuclei have the same ground state energy (taking into consideration, of course, the Coulomb energy and the difference between the neutron and proton masses), and should possess a similar system of levels. It is known about these levels that all those with E < 6 Mev have T = 1. This follows from the fact that all these levels manifest themselves in reactions $Be^9(\alpha p)B^{12}$ or B^{11} + n.

C^{13} and N^{13}

The ground state energies of these nuclei with $T_{\zeta} = \pm ½$ (minus the Coulomb energy and the difference between the neutron and proton masses) approximately coincide. Coinciding also are the moments and parities of several first excited states of these nuclei. Actually, that is how it should be, because C^{13} and N^{13} are mirror nuclei. At first glance, this does not accord with the fact that the first excited level of N^{13} is greatly downshifted as compared to the corresponding level of C^{13} (2.37 as compared to 3.08). The reason for that is understandable as, according to the shell model, C^{13} and N^{13} are a system consisting of a neutron (or proton) above the closed shells of the C^{12} nucleus. The ground state of this nucleon is on shell $1p_{3/2}$, and the transition of this nucleon to another

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* These expectations are based on the fact that it is in this state that the C^{12} nuclei are formed in the reaction $O^{16}(\gamma\alpha)C^{12}$ (see analogical reaction $C^{12}(\gamma\alpha)Be^8$ in paragraph on Be^8.)

shell, probably $2s_{1/2}$, corresponds to the first excited state of these nuclei. Such a transition, naturally, changes the average distance between the outer nucleon and the closed shells. The result is a reduction of the Coulomb energy in the first excited state of the N^{12} nucleus, as compared to the ground state, that is, a lowering of that level in N^{13} as compared to C^{13} where that effect is absent. This, incidentally, makes it possible to estimate the increase in the average nucleon radius during the transition $1p_{1/2} \to 2s_{1/2}$: $\frac{\Delta r}{r} \sim 20\%$.

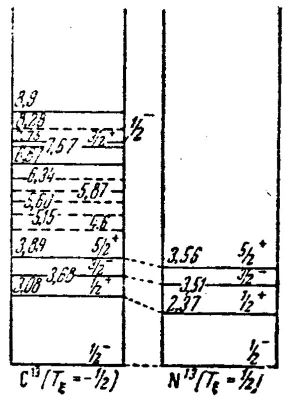


Fig. 12.

The isotopic spin of all the states of C^{13} (and, accordingly, N^{13}) with an energy of < 9 Mev is equal to ½. This follows from the fact that all these levels are manifested in reactions $C^{12}(dp)C^{13}$ or $B^{10}(\alpha p)C^{13}$. At present, the position of the first level with T = 3/2 is unknown.

C^{14}, N^{14} and O^{14}.

The $N^{14}(T_\zeta = 0)$ nucleus can be found in states with T = 0, 1, 2, ..., but states with T = 0 cannot occur in $C^{14}(T_\zeta = -1)$ and $O^{14}(T_\zeta = 1)$ nuclei. On the other hand, considerations of energy make it clear that the first excited state of N^{14} with E = 2.31 Mev corresponds to the ground state of C^{14}, so that the ground state of N^{14} should be assigned T = 0 , and the first excited state T = 1 (the ground state of C^{14} has T = 1 as is shown by reaction $C^{12}(Tp)C^{14}$). This conclusion is confirmed also by the fact[5] that, first, the N^{14} state with E = 2.31 Mev is not manifested in the reaction of inelastic scattering of deuterons on N^{14}: $N^{14}(DD')N^{14*}$, second, this state is not manifested in reaction $O^{16}(d\alpha)N^{14*}$ (as we shall see later, in O^{16}(T = 0), so that this result points directly to the isotopic spin of the N^{14} state with E = 2.31 Mev T = 1) and, finally, such an identification is indicated by the fact that the spin of this state of N^{14} ζ = 0+ is equal to the spin of the ground state of C^{14}.

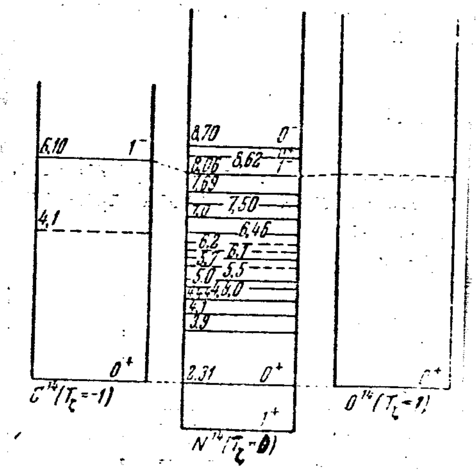


Fig. 13.

C^{14} is known to have an excited level with E = 6.10 Mev (T = 1). The level of N^{14} with E = 8.06 Mev apparently corresponds to that level. Such a deduction is based on the fact that, first, the electron-transition to the ground state of N^{14} is permitted from this level, second, the moment of this level is equal to 1-, as in the C^{14} level and E = 6.10 Mev and, finally, the energy difference between the N^{14} level with E = 8.06 Mev and the first level with T = 1 (E = 2.31 Mev) is about 6 Mev. The excited levels of N^{14} with 2.31 < E < 8.06 apparently have T = 0, as no level below 6.10 Mev has so far been found in C^{14}.

The O^{14} nucleus has been studied much less than the C^{14}, but the binding energy of these nuclei shows that their ground states are similar.

N^{15} and O^{15}

The binding energy of these nuclei and the similar arrangement of their levels show that they are mirror nuclei. It follows from reaction $N^{14}(dp)N^{15}$ that all the levels of N^{15} (and that means also O^{15}) with E < 9 Mev have T = ½. The arrangement of the first level with T = 3/2 is still not definitely known but, in view of the fact that the binding energy of $C^{15}(T_\zeta = -3/2)$ is 8.8 Mev lower than the binding energy of N^{15}, it would appear that the N^{15} level with T = 3/3 lies somewhere in the area of 11 Mev (N^{15} has a superfluous proton, in comparison with C^{15}, that is the Coulomb energy of N^{15} is by $\sim$3 Mev higher than in C^{15}, but C^{15} has a superfluous neutron which, in turn, increases the mass of that nucleus by $\sim$ 0.8 Mev. Hence we obtain the above-cited estimate: 8.8 + 3.0 - 0.8 = 11 Mev).

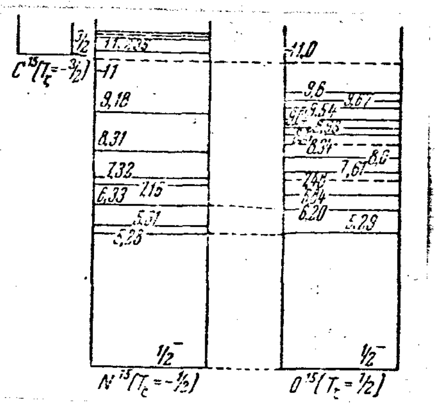


Fig. 14.

N^{16}, O^{16}, F^{16}

The $O^{16}(T_\zeta = 0)$ nucleus has a state with T = 0, 1, 2, ..., but states with T = 0 cannot occur in N^{16} and F^{16} nuclei. The binding energy of the N^{16} nucleus is by $\sim$ 10 Mev lower than in O^{16}. The ground state of O^{16} should therefore be assigned T = 0. This accords with the shell model according to which all the shells in the ground state of O^{16} are closed (the isotopic spin of a closed shell is equal to zero). The first excited state of O^{16} with T = 1 has an energy of E = 12.51 Mev[23]. That this state has T = 1 follows from the fact that the electron-transition to a ground state is possible from this level, but the decay according to scheme $O^{16} \longrightarrow C^{12} + \alpha$ is forbidden. Moreover, the moment of this level, 2-, coincides with that of the ground state of N^{16}. The fact that there are no states with T = 1 in O^{16} at lower energies follows from a comparison of the O^{16} and N^{16} binding energy. N^{16} has an excited state with an excitation energy of 1.6 Mev. A corresponding level should be found also in O^{16} but so far it has not been identified.

The F^{16} nucleus, which should be similar to N^{16}, is unstable because of the high Coulomb energy.

N^{17}, O^{17}, F^{17}

O^{17} and F^{17} are mirror nuclei with $/T_\zeta/ = ½$, and N^{17} has $T_\zeta = -3/2$. The ground states of O^{17} and F^{17}, minus the Coulomb energy and the difference between the neutron and proton masses, have the same energy, and the binding energy of N^{17} is 8.8 Mev lower than that of O^{17}. The ground states of O^{17} and F^{17} therefore have $T = ½$. The first excited level of F^{17} is somewhat down-shifted in comparison with the corresponding level of O^{17}. Just as in the case of C^{13} and N^{13}, this can be interpreted as the effect of a nucleus becoming "swollen" when excited. The isotopic spin of the ground state of N^{17} is equal to $T = 3/2$. The corresponding level of O^{17} should be found at an energy of $\sim$ 11.0 Mev, and all the lower O^{17} and F^{17} levels should have $T = ½$.

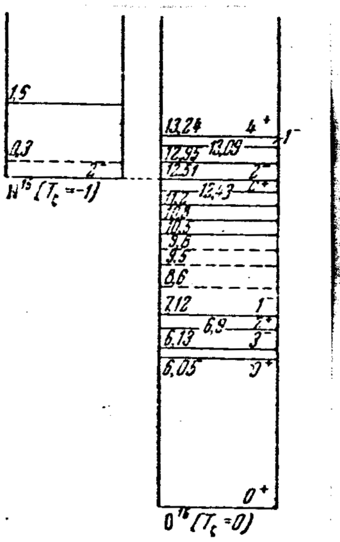


Fig. 15.

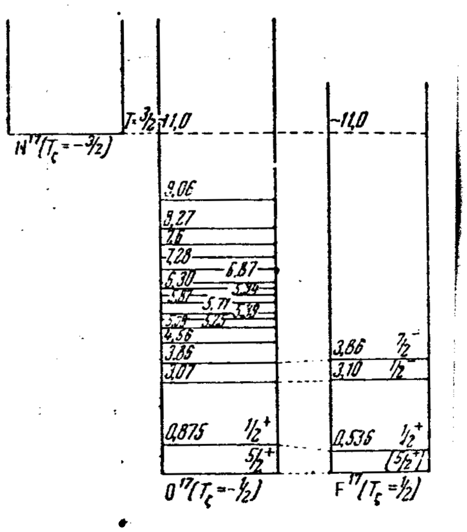


Fig. 16.

O^{18}, F^{18}

F^{18} has $T_\zeta = 0$, and O^{18} has $T_\zeta = -1$. Accordingly, states with $T = 0$ that cannot occur in O^{18} are possible in F^{18}. The isotopic spin of the ground state of O^{18} is equal to 1. This follows from the fact that, according to the nuclear shell theory, this nucleus has two neutrons above the filled O^{16} shells, and a system of two identical nucleons has $T = 1$.

Comparing the binding energy of F^{18} and O^{18}, it is easy to see that the ground state of F^{18} must have $T = 0$, and the first excited state with $T = 1$ should have an energy of $\sim$ 1 Mev. F^{18} actually has an energy level with $E = 1.05$ Mev. That level is manifested in the reaction $Ne^{20}(d\alpha)F^{18*}$. Inasmuch as the isotopic spin of Ne^{20} is T = 0, it follows from that reaction that the state of F^{18} under consideration has T = 0. However, it is possible that in this case the selection rules of the isotopic spin are violated, in view of the large quantity of admixtures, so that the isotopic spin of the excited state of F^{18} with E = 1.05 Mev is actually equal to T = 1. It may be assumed also that the F^{18} level with E = 1.05 Mev has T = 0, and that another level with T = 1 is found near it. To make a choice between these two possibilities, the moment and parity of the F^{18} level with E = 1.05 Mev should be measured.

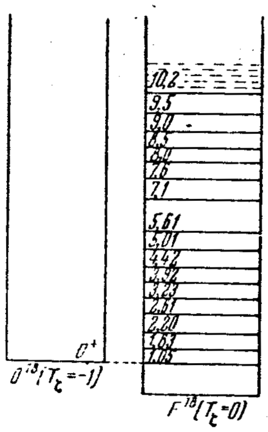


Fig. 17.

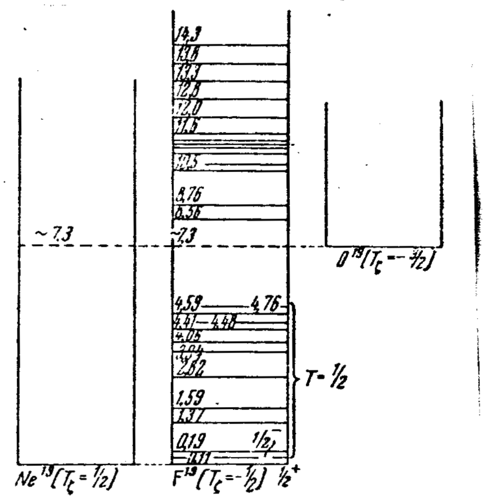


Fig. 18.

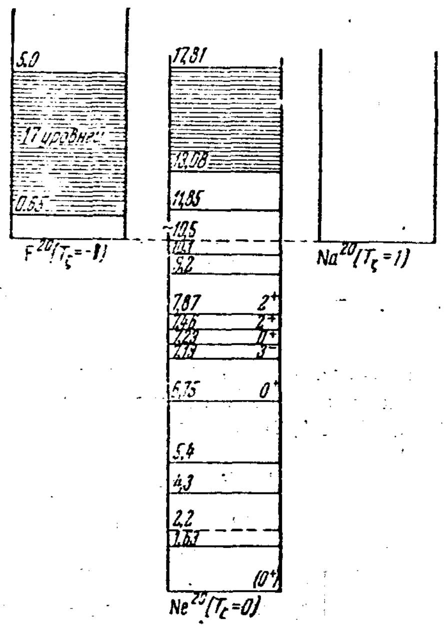
O^{19}, F^{19}, Ne^{19}

F^{19} and Ne^{19} have $/T_\zeta/ = ½$. It follows from considerations of energy that the ground states of these nuclei are similar. The binding energy of O^{19}, which has $T_\zeta = -3/2$, is 4.5 Mev lower than the binding energy of F^{19}. From this it follows that the ground states of F^{19} and Ne^{19} have T = ½, and O^{19} has T = 3/2. The latter follows from a general tendency that states with a minimal isotopic spin are most convenient energywise, and from structural considerations similar to those used in establishing the isotopic spin of O^{18} and F^{18}. We find from the difference between the binding energies of F^{10} and O^{19} that the first state with T = 3/2 should be found in F^{18} at an energy of ~ 7.3 Mev. That level has not yet been found.

In the Z $>$ 10 nuclear region, which we will now discuss, the law of the isotopic spin conservation apparently loses its meaning so that, as pointed out at the end of Chapter III, only the static nuclear properties (energy and considerations resulting from the shell model, etc.) can be used to classify the states by the isotopic spin. Since in all the cases under review the method of identification will be the same, we will illustrate it in detail by the example of F^{20}, Ne^{20} and Na^{20}, citing only the results of the identification in all the other cases.

F^{20}, Ne^{20}, Na^{20}

Ne^{20} has $T_\zeta = 0$, so that states with T = 0, 1, 2, ..., are possible, while in F^{20} and Na^{20} the T_ζ are equal to -1 and +1, respectively, and T = 0 states cannot occur in them.



уровней = levels

Fig. 19.

The F^{20} nucleus is heavier than the Ne^{20} nucleus by 7 Mev. If their states were similar, the higher Coulomb energy of Ne^{20} should have made it heavier than F^{20} by approximately 4.5 Mev. Consequently, Ne^{20} has another isotopic spin which cannot occur in F^{20}. We thus conclude that the ground state of Ne^{20} has T = 0. On the contrary, comparing the masses of F^{20} and Ne^{20}, we find that their masses differ approximately by the Coulomb energy (minus the difference between the masses of two neutrons and two protons). That means, that the two nuclei are similar. It is easy to see that they have T = 1.

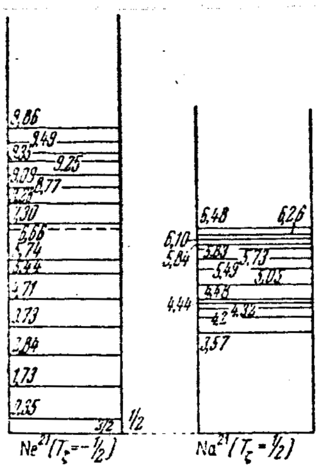


Fig. 20.

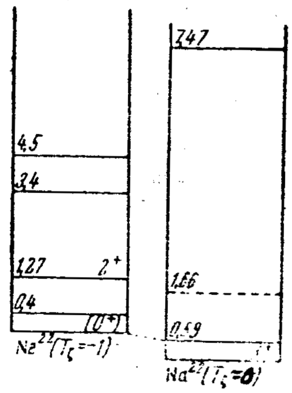


Fig. 21.

If, for example, T = 2, it would imply the existence of an O^{20}(T_ζ = -2, T = 2) nucleus which is lighter than F^{20} (due to the lower Coulomb energy). Then F^{20} should have decayed according to the scheme $F^{20}(\beta+)O^{20}$ and the O^{o} isotope would have been stable (it could have changed to Ne^{20} only by a double β-decay). Hence we conclude that the value T = 2 (and especially T > 2) is impossible for the ground state of F^{20}. Having established the isotopic spin of the ground states of F^{20} and Na^{20}, we find that the first state with T = 1 is found in Ne^{20} at an energy of ~ 10.5 Mev.

Ne^{21}, Na^{21}

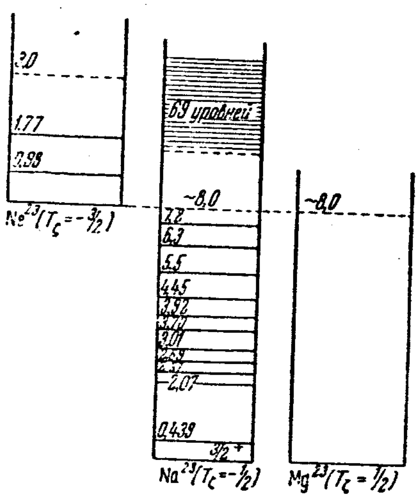
These are mirror nuclei with $/T_\zeta/$ = ½. The isotopic spin of the ground states is equal to ½. The position of the first level with T = 3/2 is not known.

Ne^{22}, Na^{22}

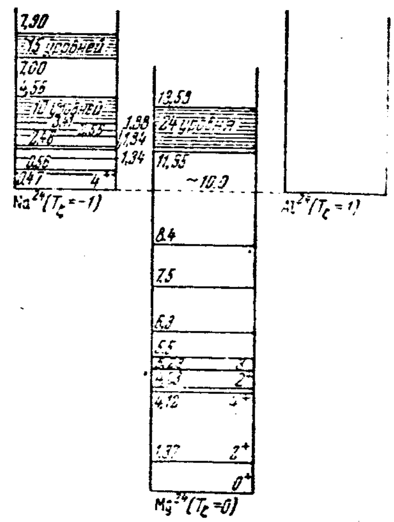
Na^{22} has T_ζ = 0, and Ne^{22} T_ζ = 1. The ground state of Na^{22} has T = 0, and the first state with T = 1 (similar to the ground state of Ne^{22}) should be found at an energy of ~ 0.6 Mev. The 0.59 Mev level of Na^{22} is found in this region. If this level has T = 1, its moment must be equal to that of the ground state of Ne^{22}, that is I = 0+. The moment of that level has not yet been measured. The excited state of Ne^{22} with an excitation energy of 1.27 Mev has, like the ground state, T = -1.

Ne^{23}, Na^{23}, Mg^{23}

Na^{23} and Mg^{23} are mirror nuclei with $/T_\zeta/ = \frac{1}{2}$, and Ne^{23} has $T = -3/2$. The ground and all the excited states of Na and Mg with an energy less than 8 Mev have $T = \frac{1}{2}$. The first state with $T = 3/2$ (similar to the ground state of Ne^{23}) should be found at an energy of ~ 8.0 Mev.



уровней = levels



уровня = level

Fig. 22.

Fig. 23.

Na^{24}, Mg^{24}, Al^{24}

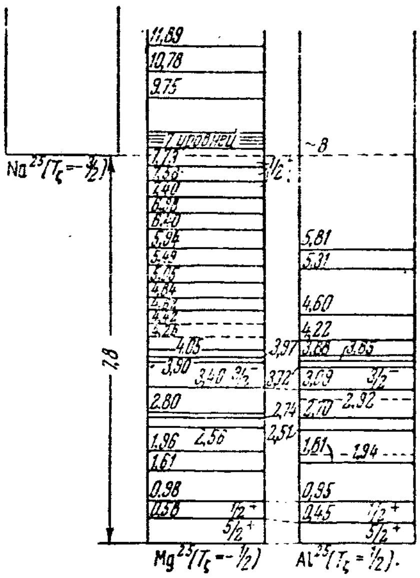
Mg^{24} has $T_\zeta = 0$, and Na^{24} and Al have $/T_\zeta/ = 1$. The ground and first excited state of Mg^{24} (up to $\sim$ 10 Mev) have $T = 0$. The first excited state with $T = 1$ (similar to the ground states of Na and Al) of Mg^{24} should be found at an energy of $\sim$10 Mev. It is possible that this is the state wherein the β-decay of Al^{24} occurs. An indication of that is the small ft of that transition (see 4, p. 105).

Na^{25}, Mg^{25}, Al^{25}

Mg^{25} and Al^{25} have $/T_\zeta/ = \frac{1}{2}$, and Na^{25} has $T_\zeta = -3/2$. Mg^{25} and Al^{25} are mirror nuclei with a similar system of levels. The ground and first excited states of these nuclei have $T = \frac{1}{2}$. The first state with $T = 3/2$ (similar to the ground state of Na^{25}) has an energy of $\sim$ 8 Mev.

Mg^{26}, Al^{26}

Al^{26} has $T_\zeta = 0$, and Mg^{26} has $T_\zeta = -1$. The ground states of Al have $T = 0$, and the first state with $T = 1$ (similar to the ground state of Mg^{26}) should be



уровней
= levels

Fig. 24.

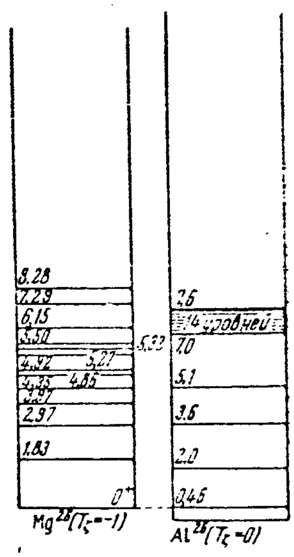
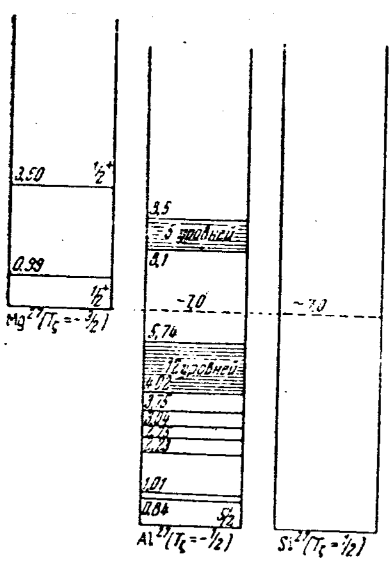


Fig. 25.

found at an energy of $\sim$ 0.6 Mev. Discovered in this region was a 0.46 Mev energy level which apparently has T = 1. This conclusion is confirmed by the radiation results of the β-decay of $Al^{26} \rightarrow Mg^{26}$ (in ft = 3.3). The spin of this state should be equal to that of the ground state of Mg^{26}, that is I = 0+.

Mg^{27}, Al^{27}, Si^{27}

Al^{27} and Si^{27} have $/T_{\zeta}/ = \frac{1}{2}$, and Mg^{27} has $T_{\zeta} = -3/2$. The ground and first excited states of Al^{27} and Si^{27} are similar and have T = ½. The first state with



Много
уровней
= Many levels

уровней
= levels

уровня
= level

Fig. 26.

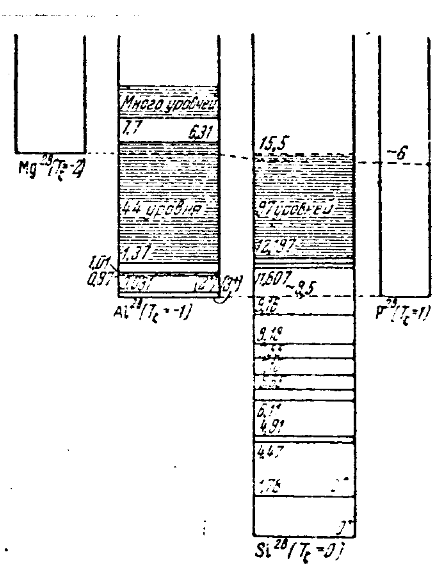


Fig. 27.

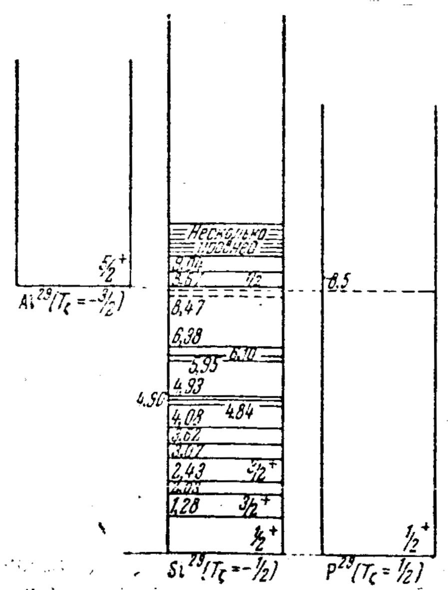
$T = 3/2$ (similar to the ground state of Mg^{27}) should be found at an energy of ~ 7 Mev, and the spin of that state should be ½+. The similar nature of the ground states of Si^{27} and Al^{27} is confirmed by the low ft value for the transition of $Si^{27} \rightarrow Al^{27}$ (ln ft = 3.6).

Mg^{28}, Al^{28}, Si^{28}, P^{28}

Si^{28} has $T_\zeta = 0$, Al^{28} and P^{28} have $/T_\zeta/ = 1$, and Mg^{28} has $T_\zeta = -2$. Accordingly, only states with $T \geqslant 2$ can occur in Mg^{28}, and $T \geqslant 1$ in Al^{28} and P^{28}. The ground and first excited states of Si^{28} have $T = 0$. The first state with $T = 1$ (similar to the ground states of Al^{28} and P^{28}) should be found at an energy of ~ 9.0 Mev. The spin of the ground state of Al^{28} is equal to (2+ or 3+). Consequently, the corresponding state of Si^{28}, as well as the ground state of P^{28}, should also have (2+ or 3+) spins. The similar nature of the ground states of P^{28} and Al^{28} is confirmed by the radiation results of the β-decay of $P^{28} \rightarrow Si^{28*}$ and $Al^{28} \rightarrow Si^{28*}$. Both of these transitions occur at the same level of Si^{28} with an excitation energy of 1.78 Mev. The first state with $T = 2$ and $I = 0+$ (similar to the ground state of Mg^{28}) should be found in Si^{28} at an energy level of ~ 15 Mev, and in Al^{28} and P^{28} at ~ 6 Mev.

Al^{29}, Si^{29} and P^{29}

Si^{29} and P^{29} have $/T_\zeta/ = ½$, and Al^{29} has $T_\zeta = -3/2$. The ground and first excited states of Si^{29} and P^{29} are similar states with $T = ½$. The first states



Несколько уровней
= several levels

Fig. 28.

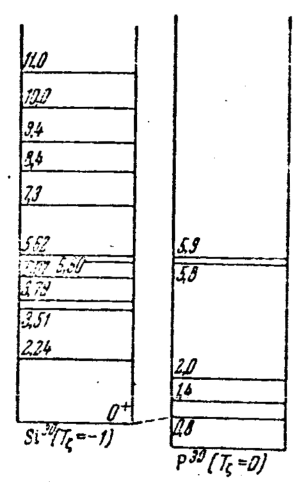


Fig. 29.

with T = 3/2 (similar to the ground state of Al^{29}) should be found at an energy level of ~ 8.5 Mev, and their spins should be equal to the ground state spin of Al^{29}, that is I = 3/2+.

Si^{30}, P^{30}

P^{30} has $T_{\zeta} = 0$, and Si^{30} has $T_{\zeta} = -1$. The ground state of P^{30} has T = 0, and that of Si^{30} has T = 1. The first state of P^{30} with T = 1, which is similar to the ground state of Si^{30}, should be found at an energy level of ~ 0.6 Mev.

An energy level of 0.8 Mev is known to exist in that region. If the isotopic spin of that level is equal to 1, its moment should be equal to that of the ground state of Si^{30}, that is I = 0+.

Si^{31}, P^{31}, S^{31}

P^{31} and S^{31} have $/T_{\zeta}/ = ½$, and Si^{31} has $T_{\zeta} = -3/2$. The ground and first excited states of P^{31} and S^{31} have T = ½. The similarity of their levels is confirmed by the low ft value of the β-transition of $Si^{31} \longrightarrow P^{31}$ (log ft = 3.6). The first states with T = 3/2 (similar to the ground state of Si^{31}) should have an excitation energy of $\sim$ 6 Mev and a spin equal to that of the ground state of Si^{31} (I = ½+ or 3/2+).

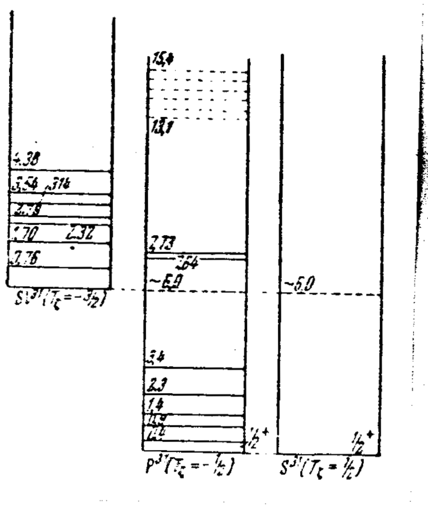


Fig. 30.

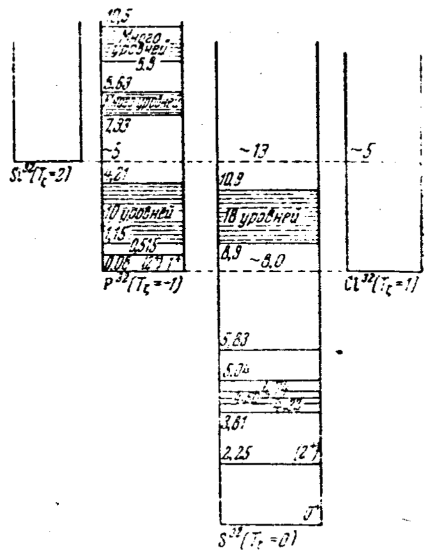


Fig. 31.

Si^{32}, P^{32}, S^{32}, Cl^{32}

S^{32} has $/T_\zeta/ = 0$, P^{32} and Cl^{32} have $/T_\zeta/ = 1$ and Si^{32} has $T_\zeta = -2$. The ground and first excited states of S^{32} have T = 0. The first state with T = 1 (similar to the ground states of P^{32} and Cl^{32}) should be found at a level of $\sim$ 8 Mev with a spin of 1+, the same as that of P^{32}. The first state with T = 2 (similar to the ground state of Si^{32}) should be found in S^{32} at a level of $\sim$13 Mev, and in P^{32} and Cl^{32} at $\sim$5 Mev.

P^{33}, S^{33}, Cl^{33}

S^{33} and Cl^{33} have $/T_\zeta/ = ½$, and P^{33} has $T_\zeta = -3/2$. S^{33} and Cl^{33} are mirror nuclei with a similar system of levels. The ground and all the low states have T = ½. The first state with T = 3/2, corresponding to the ground state of P^{33}, should have an excitation energy of $\sim$5.0 Mev.

P^{34}, S^{34}, Cl^{34}

Cl^{34} has $T_\zeta = 0$, S^{34} has $T_\zeta = -1$, and P^{34} $T_\zeta = -2$. The β-decay of $Cl^{34} \rightarrow S^{34}$ has a high probability (log ft = 3.5). That means that the ground states of S^{34} and Cl^{34} are similar, both of them therefore having T = 1, and that the first excited state (with an energy of 0.14 Mev) has T = 0 (I = 3+). The ground and first excited states of S^{34} have T = 1. Corresponding levels should be found also in Cl^{34}, particularly levels with T = 1 at energies of about 2.2 Mev (1 = 2+), 3.4 Mev and 4.9 Mev. These levels have not yet been found. The first state of Cl^{34} and S^{34} with T = 2, similar to the ground state of P^{34}, should have an excitation energy of about 10.5 Mev and a spin of 1+.

The Cl^{34} nucleus is an exception to the general empirical rule according to which the ground states of all nuclei with $T_\zeta = 0$ have T = 0[24]. But this deviation from the general rule should not be particularly surprising. Actually, Cl^{34} is an odd-odd nucleus and, according to the entire experimental material under consideration, the first states with T = 1 in odd-odd nuclei have an energy only slightly exceeding the energy of a ground state (for example, 0.46 in Al^{26}). Therefore, even a small perturbation occasioned by Coulomb interaction can lead to change in the order of levels.

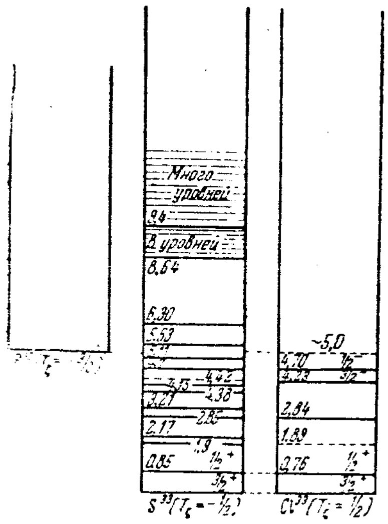


Fig. 32.

Много уровней = **many levels**

уровней = **levels**

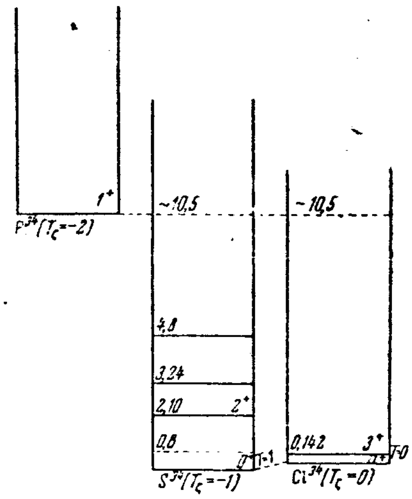


Fig. 33.

We should point out that inasmuch as the difference between the first levels with T = 0 and T = 1 in odd-odd nuclei is always decreasing with the increasing nuclear mass number, it is natural to expect the ground state of the odd-odd nuclei heavier than Cl^{34} to have T = 1, and not T = 0.

S^{35}, Cl^{35}, A^{35}

A^{35} and Cl^{35} have $/T_\zeta/ = ½$ and S^{35} has $T_\zeta = -3/2$. The ground states of A^{35} and Cl^{35} are similar and have T = ½. The first state with T = 3/2 (similar to the ground state of S^{35}) should be found at an energy of ~ 5 Mev and have a spin of 3/2. The assertion is frequently found in literature to the effect that the ground state of Cl^{35} has T = 3/2. The data on the magnetic moment of Cl^{35} are given as a basis for the assertion. If the latter were true, it would contradict the hypothesis of the charge independence of nuclear forces because the ground states of S^{35} would have had to be lower than the ground state of Cl^{35} by a magnitude equal to the excess Coulomb energy of Cl^{35}, that is by about 5 Mev, whereas actually the masses of S^{35} and Cl^{35} almost coincide. Inasmuch as the experimental material under consideration supports the hypothesis of charge independence, and even its approximate validity can hardly be doubted, it follows from our discussion that the ground state of Cl^{35} actually has T = ½.

Много уровней
= Many levels

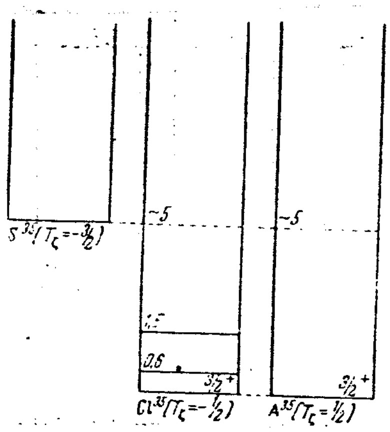


Fig. 34.

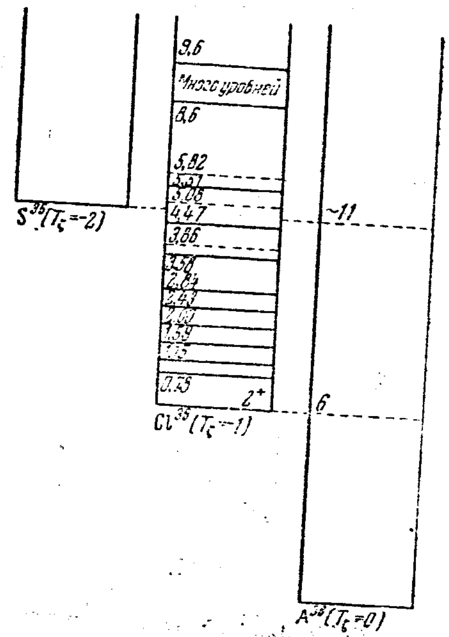


Fig. 35.

S^{36}, Cl^{36}, A^{36}

A^{36} has $T_\zeta = 0$, Cl^{36} has $T_\zeta = -1$ and S^{36} $T_\zeta = -2$. The ground state of A^{36} has T = 0, the first state with T = 1, similar to the ground state of Cl^{36}, has an excitation energy of $\sim$ 6.0 Mev and the first state with T = 2, similar to the ground state of S^{36}, has an excitation energy of $\sim$ 11 Mev. In Cl^{36} the first state with T = 2 is found at an energy of $\sim$ 5 Mev.

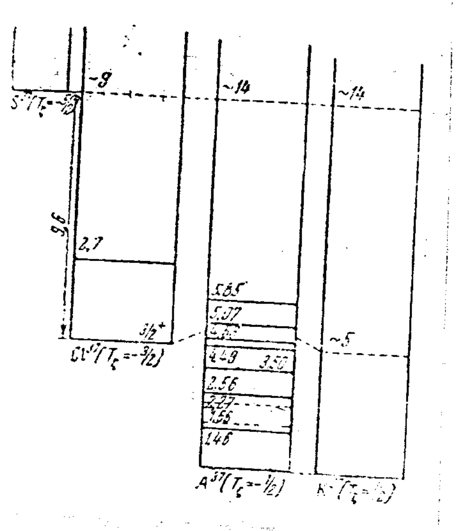
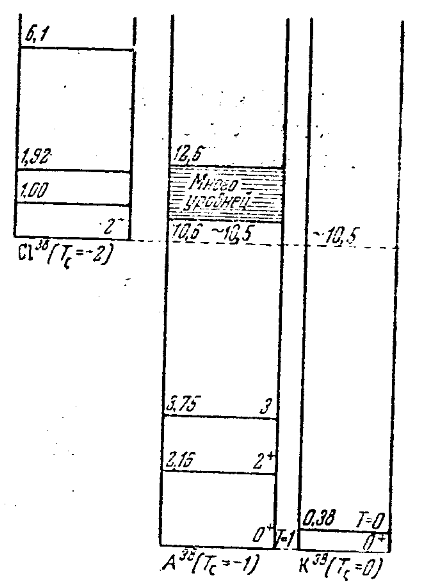


Fig. 36.



Много уровней

= Many levels

Fig. 37.

S^{37}, Cl^{37}, A^{37}, K^{37}

A^{37} and K^{37} have $/T_\zeta/ = ½$, Cl^{37} has $T_\zeta = -3/2$ and S^{37} $T_\zeta = -5/2$. The ground and first excited states of A^{37} and K^{37} are similar and have T = ½. The similar nature of these states is confirmed by the low ft magnitude (ln ft = 3.4) for the β-transition of $K^{37} \rightarrow A^{37}$. The first state with T = 3/2 (an analogue of the ground state of Cl^{37}) should have an excitation energy of ~ 5 Mev and a spin of 3/2+, and the first state with T = 5/2 (an analogue of the ground state of S^{37}) should have an energy of ~ 14 Mev. The first state of Cl^{37} with T = 5/2 should have an excitation energy of ~ 9 Mev.

Cl^{38}, A^{38}, K^{38}

K^{38} has $T_\zeta = 0$, A^{38} has $T_\zeta = -1$ and Cl^{38} $T_\zeta = -2$. The ground state of K^{38}, according to the latest experimental data, apparently has T = 1 and I = 0+, and the first excited state (E = 0.38 Mev) with I = 2+, 3+ has T = 0. If this identification is correct, then the same thing occurs in K^{38} as in Cl^{38}, that is, the energy of the first state with T = 1 becomes less than the energy of the first state with T = 0. Thus the ground states of A^{38} and K^{38} are similar. At ~ 2.2 and ~ 3.8 Mev, K^{38} should have states with T = 1 and I = 2+ and 3, respectively, similar to the corresponding states of A^{38}. The first state of A^{38} and K^{38}, which is similar to the ground state of Cl^{38}, should have an excitation energy of about 10.5 Mev and a spin of 2-.

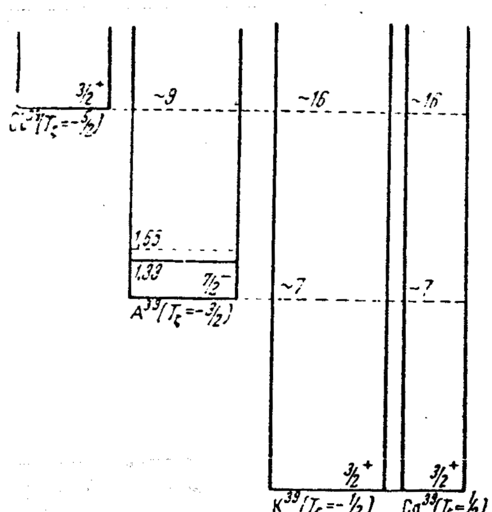


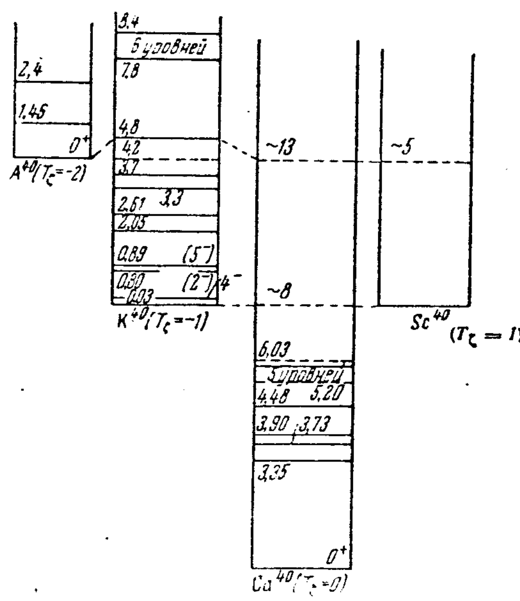
Fig. 38.

Cl^{39}, A^{39}, K^{39}, Ca^{39}

K^{39} and Ca^{39} have $/T_\zeta/ = ½$, A^{39} has $T_\zeta = -3/2$ and Cl^{39} $T_\zeta = -5/2$. The ground states of K^{39} and Ca^{39} are similar and have T = ½. The first state with T = 3/2 (an analogue of the ground state of A^{39}) of these nuclei should have an excitation energy of ~ 7 Mev and a spin of 7/2-. The first state with T = 5/2 (an analogue of the ground state of Cl^{39}) should be found at ~ 16 Mev with a spin of 3/2+. The first state of A^{39} with T = 3/2 should have an excitation energy of ~9 Mev and a spin of 3/2+.

A^{40}, K^{40}, Ca^{40}, Sc^{40}

Ca^{40} has $T_\zeta = 0$, K^{40} and Sc^{40} have $/T_\zeta/ = 1$ and A^{40} has $T_\zeta = -2$. The ground and first excited states of Ca^{40} have T = 0. The first state with T = 1 (an analogue of the ground states of K^{40} and Sc^{40}) should have an excitation energy of ~ 8 Mev and a spin of 4-, and the first state with T = 2 (an analogue of the ground state of A^{40}) should be found at an energy of ~ 12.9 Mev with a spin of 0+. The first state of K^{40} and Sc^{40} with T = 2 should be found at ~4.9 Mev energy with a spin of 0+.



уровней = levels

Fig. 39.

A^{41}, K^{41}, Ca^{41}, Sc^{41}

Ca^{41} and Sc^{41} have $/T_\zeta/ = ½$, K^{41} has $T_\zeta = 3/2$ and A^{41} has $T_\zeta = -5/2$. The ground states of Ca^{41} and Sc^{41} have T = ½, the first state with T = 3/2 (an analogue of the ground state of K^{41}) should have an excitation energy of ~ 5.0 Mev and a spin of 3/2+, and the first state with T = 5/2 (an analogue of the ground state of A^{41}) should be found at an energy of ~ 14 Mev. The first state of K^{41} with T = 5/2 should have an excitation energy of ~ 9 Mev.

A^{42}, K^{42}, Ca^{42}

Ca^{42} has $T_\zeta = -1$, K^{42} has $T_\zeta = -2$ and A^{42} has $T_\zeta = -3$. The isotope of Sc^{42} has not been found. The ground state of Ca^{42} has T = 1 and a 0+ spin. The first excited state with I = 2+ also has T = 1 and strongly resembles the similar states (T = 1, I = 2+, and about the same energy) of the following odd-odd nuclei with T = 0: Li^6, Ne^{22}, A^{38}. The first state with T = 2 (an analogue of the K^{42} ground state) should have an energy of ~9.6 Mev and a spin of 2-. The first state with T = 3 (an analogue of the A^{42} ground state) should have at least 15.6 Mev, inasmuch as it is known that the binding energy of A^{42} is less than that of K^{42}.

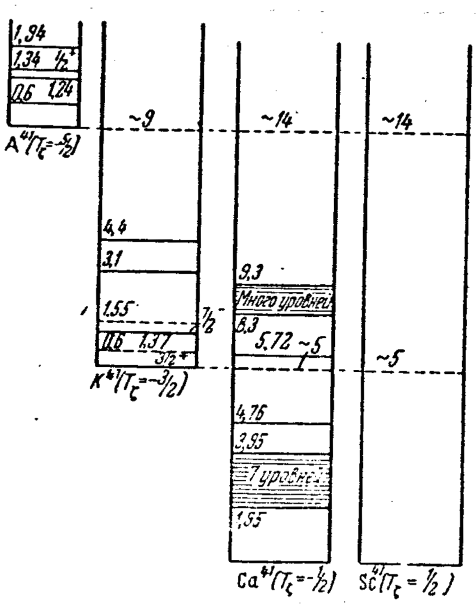


Fig. 40.

Много уровней = Many levels

уровней = levels

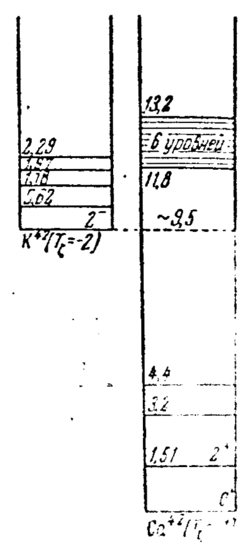


Fig. 41.

The Sc^{42} is unknown. This is the first case where a nucleus with $T_\zeta = 0$ does not exist. From this it is possible to conclude that the Coulomb energy in the nucleus has grown to such an extent as to make a direct decay $Sc^{42} \rightarrow Ca^{41} + p$ possible.

K^{43}, Ca^{43}, Sc^{43}

Sc^{43} has $T_\zeta = -½$, Ca^{43} has $T_\zeta = -3/2$ and K^{43} has $T_\zeta = -5/2$. The ground state of Sc^{43} has T = ½, the first state with T = 3/2 (an analogue of the Ca^{43} ground state) should be found at an energy of $\sim$ 4-5 Mev and have a spin of 7/2. The first state with T = 5/2 (an analogue of the K^{43} ground state) should be found in Ca^{43} at an energy of $\sim$ 5-6 Mev and in Sc^{43} at an energy of $\sim$ 10 Mev.

K^{44}, Ca^{44}, Sc^{44}*)

Sc^{44} has $T_\zeta = -1$, Ca^{44} has $T_\zeta = -2$, and K^{44} has $T_\zeta = -3$. The ground state of Sc^{44} has T = 1, the first state with T = 2 (an analogue of the Ca^{44} ground state) has an excitation energy of 3 Mev, and the first state with T = 3 (an analogue of the K^{44} ground state) has an energy of 11 Mev.

Ca^{45}, Sc^{45}, Ti^{45}

Ti^{45} has $T_\zeta = \frac{1}{2}$, Sc^{45} has $T_\zeta = -3/2$ and Ca^{45} has $T_\zeta = -5/2$. The ground state of Ti^{45} has T = ½, the first state with T = 3/2 (an analogue of the Sc^{45} ground state) has an excitation energy of $\sim$ 5 Mev, and the first state with T = 5/2 (an analogue of the Ca^{45} ground state) has an energy of $\sim$ 12 Mev.

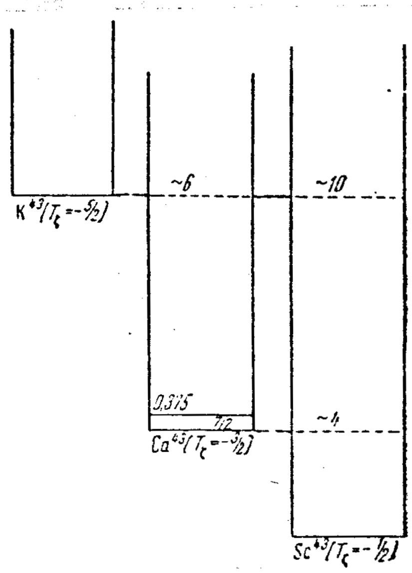


Fig. 42.

Ca^{46}, Sc^{46}, Ti^{46}

Ti^{46} has $T_\zeta = -1$, Sc^{46} has $T_\zeta = -2$ and Ca^{46} has $T_\zeta = -3$. The ground state of Ti^{46} has T = 1, the first state with T = 2 (an analogue of the Sc^{46} ground state) has an energy of $\sim$ 9 Mev, and the first state with T = 3 (an analogue of the Ca^{46} ground state) has an energy less than 16 Mev.

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*) The mass and spin differences of the further nuclei were taken from the decay schemes in the Tables[25].

Ca^{47}, Sc^{47}, Ti^{47}, V^{47}

V^{47} has $T_\zeta = -\frac{1}{2}$, Ti^{47} has $T_\zeta = -3/2$, Sc^{47} has $T_\zeta = -5/2$ and Ca^{47} has $T_\zeta = -7/2$. The ground state of V^{47} has $T = \frac{1}{2}$, the first state with $T = 3/2$ (an analogue of the Ti^{47} ground state) should have an excitation energy of ~ 6 Mev, and the first state with $T = 5/2$ (an analogue of the Sc^{47} ground state) an energy of ~ 23 Mev.

Ca^{48}, Sc^{48}, Ti^{48}, V^{48}

V^{48} has $T_\zeta = -1$, Ti^{48} has $T_\zeta = -2$, Sc^{48} has $T_\zeta = -3$, and Ca^{48} has $T_\zeta = -4$. The ground state of V^{48} has $T = 1$, the first state with $T = 2$ (an analogue of the Ti^{48} ground state) should have an energy of ~ 3 Mev, the first state with $T = 3$ (an analogue of the Sc^{48} ground state) should have an energy of ~ 14 Mev and, finally, the first state with $T = 4$ (an analogue of the Ca^{48} ground state) should have an energy of ~ 21 Mev.

Ca^{49}, Sc^{49}, Ti^{49}

Ti^{49} has $T_\zeta = -5/2$, Sc^{49} has $T_\zeta = -7/2$ and Ca^{49} has $T_\zeta = -9/2$. The first Ti level with $T = 7/2$ lies at an energy of ~ 9 Mev, and the first level with $T = 9/2$ at ~ 19 Mev.

Ti^{50}, V^{50}, Cr^{50}

Cr^{50} has $T_\zeta = -1$, V^{50} has $T_\zeta = -2$ and Ti^{50} has $T_\zeta = -3$. The ground state Cr^{50} has $T = 1$, the first state with $T = 2$ (an analogue of the V^{50} ground state) should have an energy of $\sim$ 8-9 Mev, and the first state with $T = 3$ (an analogue of the Ti^{50} ground state) should have an energy of $\sim$ 13-14 Mev.

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